ITCS 4145/5145 Assignment 2

Gravitational *N*-Body Problem

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The Problem. The objective is to find the positions and movements of bodies in space (e.g., planets) that are subject to gravitational forces from other bodies using Newtonian laws of physics. The gravitational force between two bodies of masses m_a and m_b is given by

$$F = \frac{Gm_a m_b}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies. When there are multiple bodies, each body will feel the influence of each of the other bodies and the forces will sum together (taking into account the direction of each force). Subject to forces, a body will accelerate according to Newton's second law:

$$F = ma$$

where m is the mass of the body, F is the force it experiences, and a is the resultant acceleration. All the bodies will move to new positions due to these forces and have new velocities. Written as differential equations, we have:

$$F = m\, dv/dt$$

and

$$v = dx/dt$$

where v is the velocity. For a computer simulation, we use values at particular times, t_0, t_1, t_2, and so on, the time intervals being as short as possible to achieve the most accurate solution. Let the time interval be Δt. Then, for a particular body of mass m, the force is given by

$$F = \frac{m(v^{t+1} - v^t)}{\Delta t}$$

and a new velocity

$$v^{t+1} = v^t + \frac{F\Delta t}{m}$$

where v^{t+1} is the velocity of the body at time $t + 1$, and v^t is the velocity of the body at time t. If a body is moving at a velocity v over the time interval Δt, its position changes by

$$x^{t+1} - x^t = v\Delta t$$

where x^t is its position at time t. Once bodies move to new positions, the forces change and the computation has to be repeated. The velocity is not actually constant over the time interval, Δt, so only an approximate answer is obtained.

Three-Dimensional Space. If the bodies are in a three-dimensional space, all values are vectors and have to be resolved into three directions, x, y, and z. In a three-dimensional space having a coordinate system (x, y, z), the distance between the bodies at (x_a, y_a, z_a) and (x_b, y_b, z_b) is given by

$$r = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2 + (z_b - z_a)^2}$$

The forces are resolved in the three directions, using, for example,

$$F_x = \frac{Gm_a m_b}{r^2}\left(\frac{x_b - x_a}{r}\right)$$

$$F_y = \frac{Gm_a m_b}{r^2}\left(\frac{y_b - y_a}{r}\right)$$

$$F_z = \frac{Gm_a m_b}{r^2}\left(\frac{z_b - z_a}{r}\right)$$

where the particles have the coordinates (x_a, y_a, z_a) and (x_b, y_b, z_b). Finally, the new position and velocity are computed. The velocity can also be resolved in three directions. For a simple computer solution, we usually assume a three-dimensional space with fixed boundaries. Actually, the universe is continually expanding and does not have fixed boundaries!

Sequential Code. The overall gravitational *N*-body computation can be described by the algorithm

```
for (t = 0; t < tmax; t++) {              /* for each time period */
   for (i = 0; i < N; i++) {              /* for each body */
      F = Force_routine(i);               /* compute force on ith body */
      v[i]new = v[i] + F * dt / m;        /* compute new velocity and */
      x[i]new = x[i] + v[i]new * dt;      /* new position (leap-frog) */
   }
   for (i = 0; i < N; i++) {              /* for each body */
      x[i] = x[i]new;                     /* update velocity and position*/
      v[i] = v[i]new;
   }
}
```

Task 1

Write a sequential C program that computes the movement on *N* bodies in two dimensions of a time period, where *N* is a constant. Set $N = 8$. Let the number of iterations be another constant *T*. Set $T = 100$. Choose suitable masses, initial positions and velocities and store in an array, i.e.

Table 1: Input data

Body	Mass	Position in x direction	Position in y direction	Velocity in x direction	Velocity in y direction
0					
1					
2					
3					
4					
5					
6					
7					

Update this array after each iteration and display the values.

Task 2

Find a way to display the movement of the bodies graphically. Possible ways will be given in a separate document.

Task 3

Re-write your program as an MPI program to execute on coit-grid05.uncc.edu. (coit-grid05.uncc.edu has 4 processors and 16 cores.) Incorporate code to measure the time of execution. Use the following approach:

1. The master process holds the array and broadcast the array to the slaves
2. Each slave presents one body and computes the new position for that body
3. All positions are gathered by the master process and the array is updated
4. Steps 1 - 3 are repeated

Undergraduates (ITCS 4145) - May work in pairs. Submit one document with both names given

Graduates (ITCS 5145) - May work in pairs. Submit one document with both names given.

Assignment Submission

Produce a document that show that you successfully followed the instructions and performs all tasks by taking screen shots and include these screen shots in the document. Include insightful conclusions. To include screen shots from Windows XP, select window, press Alt-Printscreen, and paste to file. Submit to Blackboard by the due date (see home page). **Include all code.**