

Technical Section

Linear approximation of Bidirectional Reflectance Distribution Functions

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Abstract

Various empirical and theoretical models of the surface reflectance have been introduced so far. Most of these models are based on functions with non-linear parameters and therefore faces some computational difficulties involved in non-linear optimization processes. In this paper, we introduce a new approach for approximating Bidirectional Reflectance Distribution Functions (BRDF) by employing response surface methodology. The proposed model employs principal component transformations of the explanatory variables which are essentially functions of incoming and outgoing light directions. The resulting model is linear and can be used to represent both isotropic and anisotropic reflectance for diffuse and glossy materials. Considering some widely used reflection models including the Ward model, the Ashikhmin–Shirley model and the Lafortune model, we demonstrate empirically that satisfactory approximations can be made by means of the proposed general, simple and computationally efficient linear model.

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1. Introduction

Building a comprehensive reflection model that describes the interactions between light and materials is a fundamental problem in computer graphics. To synthesize a realistic image of a scene, a complete description of reflectance is required for each surface in the scene. A class of functions called Bidirectional Reflectance Distribution Functions (BRDF) has been widely used to describe the surface reflectance [1].

Theoretically the BRDF is a function of a number of factors including incident light direction, reflected light direction, the wavelength and the surface position. Various models ranging from empirical models to physically based models for BRDFs have been developed to approximate surface reflectance. Each model can have better approximation over the others under certain conditions. Whatever

model is selected for a certain application, one needs to determine its unknown parameters. A common approach for determining the model parameters is to estimate them from the experimental data based on the BRDF measurements. Mostly the least squares techniques are used for estimating the model parameters. However, most of the models are defined by some non-linear functions and estimating the underlying parameters is not straightforward.

There are several shortcomings in model fitting when a BRDF is represented by a non-linear function [2]. One major problem is that non-linear least squares estimation requires employing some optimization algorithms. Depending on the number of lobes used in modeling the BRDF, the corresponding number of parameters to be estimated usually is large. For example, when Lafortune et al. [3] model is chosen to fit an isotropic material, there would be at least three and six non-linear parameters which should be estimated for one lobe and two lobe representations, respectively. Furthermore, optimization results closely depend on the choice of initial values of the non-linear parameters and a global minimum usually is not guaranteed. Also computational cost of estimating the non-linear parameters become high when large data set is used.

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In this paper, we introduce a representation based on response surface models. Assuming a simplified BRDF model can be expressed as functions of some variables which are essentially defined to be functions of incoming and outgoing direction vectors of the light only, we propose to transform the variables of the underlying non-linear reflectance model into an orthogonal system. The new system has the same number of variables, namely the principal components as the original system. Considering these orthogonal explanatory variables, response surface models are used to approximate the corresponding BRDF. The resulting model is linear and can be used to represent both isotropic and anisotropic diffuse and glossy reflections. Weighted least squares technique is used to estimate the model parameters. Based on a data set, we demonstrate that satisfactory approximations to BRDF of a given diffuse and glossy surfaces can be made by means of the proposed general, simple and computationally efficient model for which no non-linear estimation is involved.

The next section gives a review of previous work. In Section 3, the response surface model based on principal components approach is described and weighted least squares estimation of the corresponding model is introduced in Section 4. Error of approximation for the proposed model is given in Section 5. Some empirical results are presented in Section 6 and Section 7 is devoted to discussions.

2. Previous work

Although there has been an extensive literature on modeling surface reflectance, developing BRDF models still is an active area of research. A good treatment of the subject may be found in Ref. [4].

The most well known and one of the oldest reflectance model developed to simulate the effects of the specular reflection is the Phong model [5]. For specular surfaces, this model assumes that the incoming light tends to bounce off in the reflection direction. Fresnel effect and micro geometry of a surface are important factors in BRDF modeling. Considering the fact that a surface can be modeled by using small and flat surfaces (*microfacets*) with random angle and size, Ashikhmin et al. [6] and Cook–Torrance [7] have presented more sophisticated models. A comprehensive but computationally expensive model based on physical theory was developed by He et al. [8]. Oren and Nayar [9] presented a non-Lambertian diffuse model to simulate the surfaces such as sand and plaster. For anisotropic surfaces, Poulin and Fournier [10] introduced a reflection and refraction model.

The BRDF technology has been extensively utilized in computer graphics and a wide range of physically based models have been developed along this line. Using these models is problematic since the corresponding parameters are not readily available and their functional forms present some complexity [11].

Instead of computing a BRDF from a theoretical model it might be more convenient to measure the reflectance

properties of the sample for certain cases [12]. Ward [13] developed a method for acquiring BRDFs and introduced a model to represent the reflectance of anisotropic surfaces. A simple method for representing the BRDFs of a given surface would be to sample the BRDF at various points over the surface and apply an interpolation whenever is needed. However, storage problems of the large data set, existence of noise and missing observations are the limiting factors for the practical applications of this approach. He et al. [14] presented a hybrid representation based on using spline patches.

All theoretical models can be fitted to a given data by using non-linear estimation techniques [4]. On the other hand, empirical models which do not necessarily reflect the physical mechanism of the light material interaction have been used to capture the reflectance effects [13,15]. Lafortune et al. [3] introduced an empirical model based on a class of primitive functions with non-linear parameters providing a simple and compact representation. Stark et al. [16] proposed new sets of coordinates which are barycentric with respect to a triangular support to reduce the dimensionality of several BRDF models.

A number of models based on representing BRDF through linear functions have also been developed. Westin et al. [17] used spherical harmonics to store BRDF data. Schröder and Sweldens [18] have reported some results in storing BRDFs by using wavelets. Koendrink et al. [19] used Zernike polynomials for representing BRDFs in terms of orthonormal basis functions to map the points on a hemisphere onto a disk. Matusik et al. [2] employed the principal component analysis to extract a relatively small number of basis functions to span the entire space. Kautz and McCool [20] have represented the BRDF as a sum of two-variable separable functions. A common drawback of these models is that the number of parameters to be estimated is large.

3. Approximation of BRDF model

In this paper, we employ the well-known *response surface* models with linear parameters to approximate the BRDF.

Response surface methodology is widely used to explore the relationships between multiple explanatory variables and one dependent variable. A response surface model is defined as a polynomial function of order p in k variables [21,22]. For example, a second-order response surface model in two variables ($p=2$, $k=2$) may be written as

$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_1 x_2 + \alpha_4 x_1^2 + \alpha_5 x_2^2 + e, \quad (1)$$

where e represents the error term. This model reflects linear, quadratic and interaction effects on the response variable y . If the explanatory variables x_1 and x_2 are orthogonal then the corresponding coefficient for the interaction term becomes zero and removed from the model.

For a specified surface point at which the corresponding BRDF was measured, a response surface model of degree p with k variables may be written in terms of

these variables as

$$\begin{aligned}
 y = & \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \dots + \alpha_k X_k + \alpha_{k+1} X_1^2 \\
 & + \alpha_{k+2} X_2^2 + \dots + \alpha_{2k} X_k^2 + \dots + \alpha_{(p-1)k+1} X_1^p \\
 & + \alpha_{(p-1)k+2} X_2^p + \dots + \alpha_{kp} X_k^p + \dots \\
 & + \text{(first order interactions)} \\
 & + \text{(higher order interactions)} + e. \tag{2}
 \end{aligned}$$

It may be noted that the above model is linear in all parameters. The number of terms in the model increases exponentially depending on the degree of the polynomial. One way of simplifying this model is to transform k -dimensional system into an orthogonal system. If the hyper surface is defined in terms of the orthogonal variables then the first-order interaction terms in Eq. (2) will be zero since the new explanatory variables are uncorrelated. Obviously the higher-order interaction terms will not disappear from the model even if the corresponding variables are uncorrelated. However, one may expect that the higher-order interaction effects will be reduced considerably when the underlying data space is transformed into an orthogonal basis.

Based on the fact that every linear regression model can be restated in terms of orthogonal variables which are also linear functions of the explanatory variables, any orthogonal transformation could be used for this purpose. We chose to use principal components as the orthogonal transforms of the explanatory variables. Although such linear transformation lacks simple interpretation since each of the transformed variables are the mixture of the original variables, the new variables provide a unified approach for estimating the model parameters [23–25].

The principal component analysis is mostly used for reducing multidimensional measurement variables to a smaller set of orthogonal variables [26]. The first principal component accounts for a maximum amount of the total variation represented in the complete set of original variables. The second principal component accounts for a maximum amount of the remaining variation, etc. This property of principal components opens a way to represent the BRDF surfaces in reduced dimensions.

Let Z_1, Z_2, \dots, Z_p be the principal component transformations of the respective variables X_1, X_2, \dots, X_p . Using these orthogonal principal components the corresponding BRDF surface can be approximated by the following simplified model

$$\begin{aligned}
 f = & \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \dots + \beta_k Z_k + \beta_{k+1} Z_1^2 \\
 & + \beta_{k+2} Z_2^2 + \dots + \beta_{2k} Z_k^2 + \dots + \beta_{(p-1)k+1} Z_1^p \\
 & + \beta_{(p-1)k+2} Z_2^p + \dots + \beta_{kp} Z_k^p + e, \tag{3}
 \end{aligned}$$

where $\beta_i, (i = 0, 1, \dots, 3p)$ are the parameters and e is the error term with mean zero and constant variance σ^2 . The proposed polynomial function in Eq. (3) provides a flexible model for representing complex BRDF surfaces. The parameters of the model can be easily estimated from the measured BRDF data by using standard linear

regression methods. As opposed to non-linear models, there is no optimization technique required to obtain the parameter estimates when this model is used. However, the use of this model is limited to diffuse and glossy materials since representing highly specular materials may require employing polynomial models of prohibitively high degrees.

Choosing the most convenient set of explanatory variables for the polynomial model is not a trivial problem. In a typical application, one can consider the spherical coordinates of incoming and outgoing direction vectors as explanatory variables such that $X_1 = \theta_o, X_2 = \phi_o, X_3 = \theta_i, X_4 = \phi_i$ where θ_i and θ_o are the elevation angles; ϕ_i and ϕ_o are the azimuth angles of incoming and outgoing directions, respectively. Another possibility is to use halfway vector representation which has been used in many recent BRDF models [27]. In this section, we proceed to define the explanatory variables as functions of some direction angles that have been employed by three widely used reflection models, namely the Ward model, the Ashikhmin–Shirley model and the Lafortune model and demonstrate the application of our approach. Following the similar approach as in Ref. [27] the diffuse contribution was forced to Lambertian in these three reflection models.

3.1. The Ward model

The original Ward [13] model for isotropic materials can be written as

$$f = \mu + \rho \frac{1}{\{(\mathbf{n} \cdot \mathbf{u})(\mathbf{n} \cdot \mathbf{v})\}^{1/2}} \frac{\exp\{-\tan^2 \delta / \alpha^2\}}{4\pi\alpha^2}, \tag{4}$$

where μ, ρ and α are the model parameters, \mathbf{n}, \mathbf{u} and \mathbf{v} are the normal, incoming and outgoing vectors, respectively, and δ is the angle between \mathbf{n} and the halfway vector $\mathbf{h} = (\mathbf{u} + \mathbf{v}) / \|\mathbf{u} + \mathbf{v}\|$. Noting that

$$\tan^2 \delta = \frac{\sin^2 \delta}{\cos^2 \delta} = \frac{1}{(\mathbf{n} \cdot \mathbf{h})^2} - 1, \tag{5}$$

the Ward model can be rewritten as

$$f = \mu + \rho^* \frac{1}{\{(\mathbf{n} \cdot \mathbf{u})(\mathbf{n} \cdot \mathbf{v})\}^{1/2}} \exp\left\{-\frac{1/(\mathbf{n} \cdot \mathbf{h})^2 - 1}{\alpha^2}\right\}, \tag{6}$$

where $\rho^* = \rho / (4\pi\alpha^2)$.

Based on this representation, we now define the explanatory variables as $X_1 = \mathbf{n} \cdot \mathbf{h}$ and $X_2 = (\mathbf{n} \cdot \mathbf{u})(\mathbf{n} \cdot \mathbf{v})$. Note that the reciprocity property is maintained with these new variables. Let Z_1 and Z_2 be the principal component transformations of the respective variables X_1 and X_2 . Using these orthogonal principal components the BRDF surface can be approximated by the following simplified polynomial model of degree p .

$$\begin{aligned}
 f = & \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_1^2 + \beta_4 Z_2^2 + \dots + \beta_{2p-1} Z_1^p \\
 & + \beta_{2p} Z_2^p + \varepsilon, \tag{7}
 \end{aligned}$$

where β_i , ($i = 0, 1, \dots, 2p$) are the parameters and ε is the error term with mean zero and constant variance σ^2 .

3.2. The Ashikhmin–Shirley model

The Ashikhmin–Shirley model [28] used in this study is defined as

$$f = \mu + \rho \frac{(\mathbf{n} \cdot \mathbf{h})^\gamma}{(\mathbf{v} \cdot \mathbf{h}) \max\{\mathbf{n} \cdot \mathbf{u}, \mathbf{n} \cdot \mathbf{v}\}} \text{Fresnel}(F_0, \mathbf{v} \cdot \mathbf{h}), \quad (8)$$

where μ , ρ , γ and F_0 are the model parameters and the vectors \mathbf{n} , \mathbf{u} , \mathbf{v} and \mathbf{h} are defined as in Eq. (4). We define the explanatory variables in such a way that the reciprocity property of the underlying model is not violated. For the

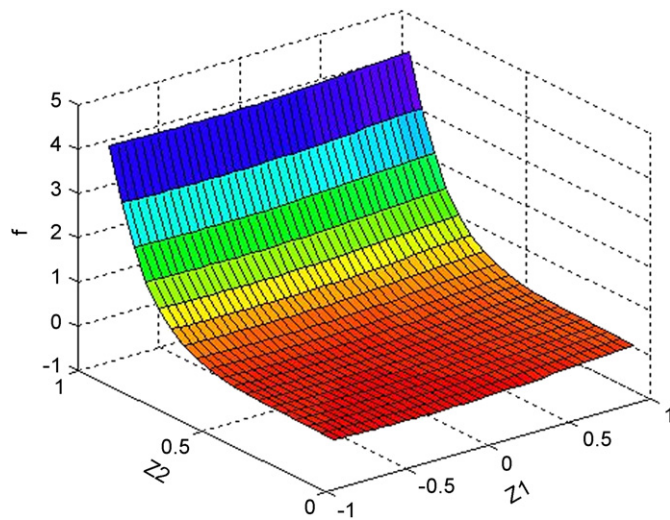


Fig. 1. Fitted isotropic BRDF surface as a function of two principal components (Cornell mystic lacquer).

above simplified model, we proceed to choose the following three variables:

$$\begin{aligned} X_1 &= \mathbf{n} \cdot \mathbf{h}, \\ X_2 &= \mathbf{v} \cdot \mathbf{h}, \\ X_3 &= \max\{\mathbf{n} \cdot \mathbf{u}, \mathbf{n} \cdot \mathbf{v}\}. \end{aligned} \quad (9)$$

The principal component transformations Z_1 , Z_2 and Z_3 of the respective variables X_1 , X_2 and X_3 are then used in Eq. (3) to obtain a polynomial model in three variables.

3.3. The Lafortune model

The Lafortune et al. [3] representation which satisfies the reciprocity property is given by

$$f = \mu + \{\alpha_x(u_x v_x) + \alpha_y(u_y v_y) + \alpha_z(u_z v_z)\}^\gamma, \quad (10)$$

where (u_x, u_y, u_z) and (v_x, v_y, v_z) are the components of the incoming and outgoing direction vectors \mathbf{u} and \mathbf{v} , respectively; μ , α_x , α_y , α_z and γ are the model parameters. The direction vectors \mathbf{u} and \mathbf{v} are defined with respect to local coordinate system at the surface location. The corresponding coordinate system is transformed in such a way that the z -axis is aligned to the surface normal and, x and y axes to the principal directions of anisotropy.

For the Lafortune model, $\alpha_x = \alpha_y$ if the reflection is isotropic, in other words, it is invariant with respect to rotation of the surface around its normal. In this special case one can define new variables as $X_1 = (u_x v_x + u_y v_y)$ and $X_2 = (u_z v_z)$ and then obtains the corresponding principal component transformations Z_1 and Z_2 . A sample isotropic BRDF surface as a function of the first and second principal components is illustrated in Fig. 1.

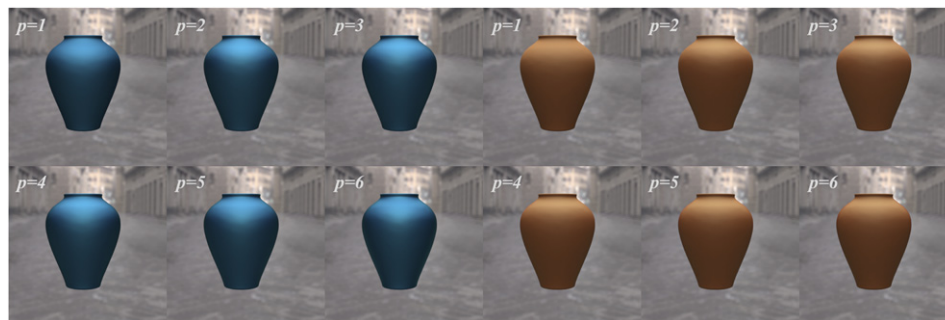


Fig. 2. Vases rendered using polynomial models (based on the Lafortune) of various degrees. Left: Cornell “Cayman lacquer”. Right: CUReT “Quarry tile”.

Table 1
L2 errors and the number of parameters used in polynomial models (based on the Lafortune) in Fig. 2

Material	$p = 1$		$p = 2$		$p = 3$		$p = 4$		$p = 5$		$p = 6$	
	L2	#Par.	L2	#Par.	L2	#Par.	L2	#Par.	L2	#Par.	L2	#Par.
Cayman lacquer	2.1983	7	2.1981	9	2.1981	11	2.1979	13	2.1972	15	2.1953	17
Quarry tile	0.0324	7	0.0205	9	0.0142	11	0.0116	13	0.0106	15	0.0093	17

4. Parameter estimation

As was stated above, a major problem in the application of the non-linear BRDF models is the difficulty in estimating their parameters. The quality of the fit depends on a good starting guess. The proposed model in Eq. (3) is linear in parameters and therefore multiple regression techniques, which do not require any initial guess for the parameters, can be directly used to obtain the least squares estimates of the underlying parameters. The total number of parameters to be estimated is $kp + 1$. Another difficulty in fitting BRDF models is the definition of the objective function. The BRDF measurements are prone to outliers especially in the region where incoming or outgoing directions close to extreme grazing angles. In general, the BRDF measurements obtained in this region are not reliable [27]. The least squares fitting is known to be sensitive to outliers because of the squaring the terms in the objective function magnifies the error contribution of these extreme values. To improve the fit of BRDF models, a common approach has been to apply the weighted least squares technique.

In this work, we adopted the following objective function proposed by Ngan et al. [27]

$$\varphi(\boldsymbol{\tau}) = \left[\frac{\sum_{i=1}^n w_i [\cos \theta_i \{b_i - f(\mathbf{u}_i, \mathbf{v}_i; \boldsymbol{\tau})\}]^2}{\sum_{i=1}^n w_i} \right]^{1/2}, \quad (11)$$

where $\boldsymbol{\tau}$ is the parameter vector of the model, b_i , \mathbf{u}_i and \mathbf{v}_i stands for the i th measured BRDF sample of size n , and the corresponding incoming and outgoing light direction vectors, respectively, w_i is the i th solid angle correction term. This metric for estimating the parameters of a model is known as the L2 metric in statistical literature. Following the work of Ngan et al. [27], we eliminated the measured BRDF values with incoming or outgoing angles larger than 80° to avoid possible outliers when estimating the parameters.

5. Approximation errors

It is obvious that the polynomial models do not meet the energy conservation property of BRDF. If any selected model provides a good representation for a BRDF surface then the corresponding polynomial model should also reflect a similar behavior. However, the polynomial model used is an approximation to this BRDF surface and therefore it encounters some errors. This error created by the model inadequacy can be used to explain the deviation from the true model.

The least squares estimators of the parameters in Eq. (3) are given by

$$\hat{\boldsymbol{\beta}} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{f}, \quad (12)$$

where \mathbf{f} is an $n \times 1$ vector of measured BRDFs, $\hat{\boldsymbol{\beta}}$ is a $(kp + 1) \times 1$ vector, \mathbf{Z} is an $n \times (kp + 1)$ design matrix whose i th row ($i = 1, 2, \dots, n$) is

$$\mathbf{z}_i = \{1Z_{1i}Z_{2i} \cdots Z_{ki} \cdots Z_{1i}^p, Z_{2i}^p \cdots Z_{ki}^p\}, \quad (13)$$

and $Z_{1i}, Z_{2i}, \dots, Z_{ki}$ are the i th principal components of the explanatory variables. If the model in Eq. (3) is a true model for representing the BRDF measurements then $\hat{\boldsymbol{\beta}}$ is

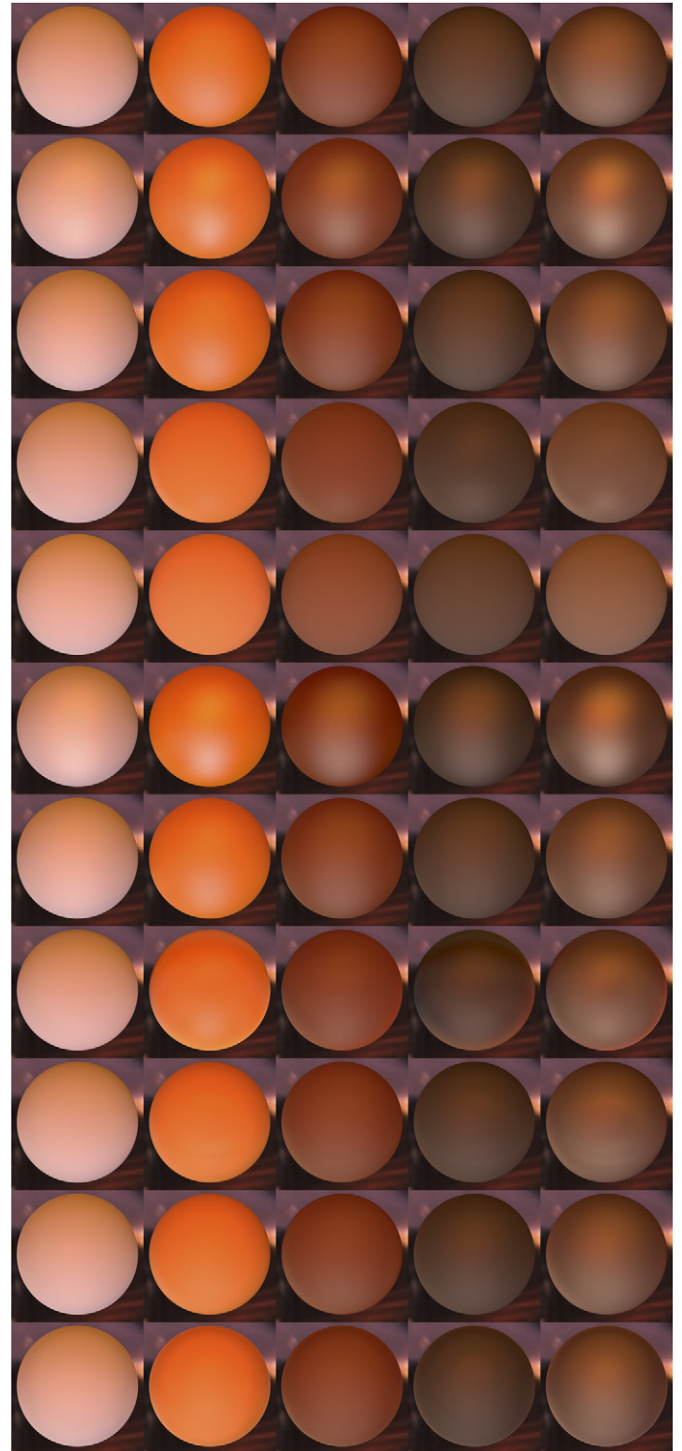


Fig. 3. Spheres rendered using nine analytical models. Top to bottom: Ashikhmin–Shirley, Blinn–Phong, Cook–Torrance, Lafortune, Oren–Nayar, Ward, Ward–Duer, Zernike polynomials (order = 8), Polynomial model (Ashikhmin–Shirley, $p = 5$), Polynomial model (Lafortune, $p = 7$), and Polynomial model (Ward, $p = 7$). Five isotropic materials are chosen (based on CURET). Left to right: Felt, Orange peel, Quarry tile, Slate_a, Slate_b.

an unbiased estimate of β [29]. However, the model itself in Eq. (3) is an approximation of the true model. The mean squared error for the predicted model evaluated at \mathbf{z} , may be expressed as

$$\begin{aligned} MSE &= E\{f(\mathbf{z}, \hat{\beta}) - \eta(\mathbf{z}, \alpha)\}^2 \\ &= V\{f(\mathbf{z}, \hat{\beta})\} + \{E\{f(\mathbf{z}, \hat{\beta})\} - \eta(\mathbf{z}, \alpha)\}^2 \\ &= \mathbf{z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{z}'\sigma^2 + \{\mathbf{z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\eta(\mathbf{z}, \alpha) - \eta(\mathbf{z}, \alpha)\}^2, \end{aligned} \quad (14)$$

where E and V stand for the expectation and variance operators, respectively, $f(\mathbf{z}, \hat{\beta})$ is the estimated model, η is the true model, α is the parameter vector of the true model and

$$\{\eta(\mathbf{z}, \alpha)\} = \{\eta(\mathbf{z}_1, \alpha), \eta(\mathbf{z}_2, \alpha), \dots, \eta(\mathbf{z}_n, \alpha)\}'. \quad (15)$$

As may be seen from Eq. (14) the approximation error is determined by the second term which is known as the *bias term* of the fitted model. Maximum approximation error as

Table 2

L2 errors of various models based on five isotropic materials in Fig. 3

BRDF model	Felt	Orange peel	Quarry tile	Slate_a	Slate_b
Ashikhmin–Shirley	0.0190	0.0217	0.0165	0.0121	0.0223
Blinn–Phong	0.0518	0.0626	0.0369	0.0196	0.0586
Cook–Torrance	0.0229	0.0213	0.0136	0.0107	0.0236
Lafortune	0.0170	0.0245	0.0117	0.0070	0.0184
Oren–Nayar	0.0571	0.0740	0.0429	0.0249	0.0711
Ward	0.0385	0.0420	0.0241	0.0124	0.0383
Ward–Duer	0.0238	0.0219	0.0137	0.0109	0.0243
Zernike (order = 8)	0.0087	0.0126	0.0069	0.0059	0.0068
Poly. (Ashikhmin–Shirley, $p = 5$)	0.0090	0.0206	0.0098	0.0063	0.0114
Poly. (Lafortune, $p = 7$)	0.0126	0.0227	0.0093	0.0062	0.0165
Poly. (Ward, $p = 7$)	0.0228	0.0277	0.0122	0.0092	0.0266

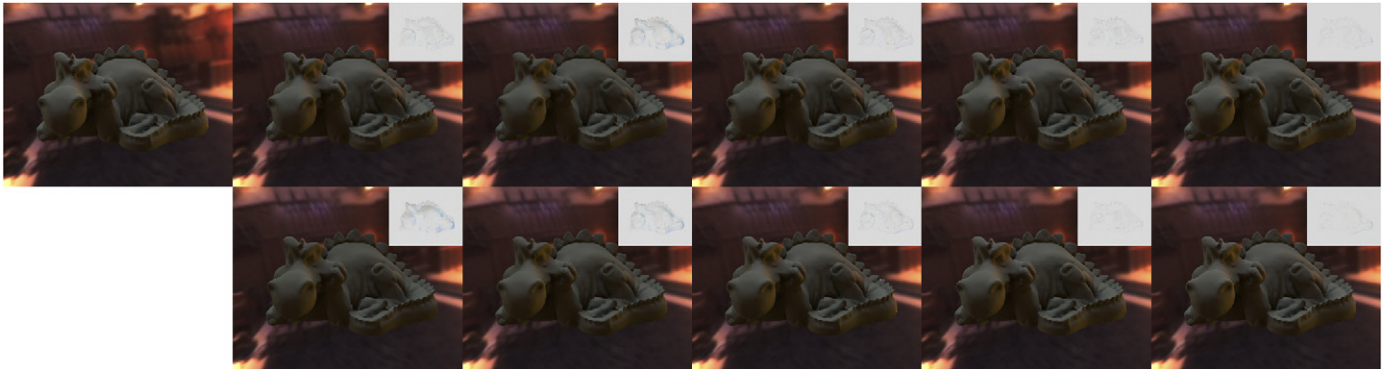


Fig. 4. Phlegmatic Dragon Figure. Fitting the measured “green-latex” [2] BRDF with the eight models. From left to right; top row: measured, Ashikhmin–Shirley, Blinn–Phong, Cook–Torrance, Lafortune, Polynomial model (Ashikhmin–Shirley, $p = 3$). Bottom row: Oren–Nayar, Ward, Ward–Duer, Polynomial model (Lafortune, $p = 5$), Polynomial model (Ward, $p = 5$).

Table 3

L2 errors and PSNR values of various models based on six isotropic materials (based on data from Ref. [2])

BRDF model	Beige-fabric		Blue-metallic-paint		Dark-blue-paint		Green-latex		Orange-paint		Yellow-plastic	
	L2	PSNR	L2	PSNR	L2	PSNR	L2	PSNR	L2	PSNR	L2	PSNR
Ashikhmin–Shirley	0.0053	46.043	0.0150	39.043	0.0043	42.330	0.0040	43.286	0.0038	46.710	0.0071	39.321
Blinn–Phong	0.0104	37.683	0.0228	31.570	0.0109	34.671	0.0064	38.841	0.0100	39.573	0.0143	36.288
Cook–Torrance	0.0080	42.629	0.0172	39.589	0.0035	43.425	0.0037	43.255	0.0037	46.789	0.0068	39.544
Lafortune	0.0075	42.113	0.0132	38.099	0.0056	42.353	0.0036	44.434	0.0061	45.504	0.0086	39.709
Oren–Nayar	0.0105	37.706	0.0540	26.654	0.0129	33.120	0.0076	38.507	0.0119	38.228	0.0153	35.654
Ward	0.0101	38.055	0.0146	34.977	0.0091	36.824	0.0052	40.540	0.0085	41.683	0.0130	36.976
Ward–Duer	0.0093	38.563	0.0171	38.899	0.0056	39.619	0.0037	43.249	0.0059	41.680	0.0103	37.663
Poly. (Ashikhmin–Shirley, $p = 3$)	0.0038	48.104	0.0331	34.291	0.0064	42.514	0.0029	47.707	0.0049	47.429	0.0082	43.864
Poly. (Lafortune, $p = 5$)	0.0056	45.083	0.0314	35.535	0.0060	42.472	0.0029	46.507	0.0049	47.244	0.0089	41.809
Poly. (Ward, $p = 5$)	0.0064	45.574	0.0347	31.333	0.0040	41.873	0.0026	49.557	0.0039	45.523	0.0061	43.561

a function of the bias term over the sample space may be expressed as

$$\delta_{\max} = \max_{1 \leq i \leq n} \{z_i(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\eta(\mathbf{z}, \boldsymbol{\alpha}) - \eta(\mathbf{z}_i, \boldsymbol{\alpha})\}^2. \quad (16)$$

For a completely specified true model and the design matrix \mathbf{Z} , calculating the corresponding maximum approximation error is straightforward.

6. Results

To investigate some empirical properties of the proposed model we have used the CURET database [30] which contains BRDF measurements for red, green and blue color channels and for each of the 61 different materials.

Measurements are obtained for each of the 205 combinations of incoming and outgoing angles. We have also used sample data sets of seven different isotropic materials “Cayman lacquer” from Cornell University data base [31], and “beige-fabric”, “blue-metallic-paint”, “dark-blue-paint”, “green-latex”, “orange-paint”, “yellow-plastic” from Matusik et al. [2] based on 1439 and 1,458,000 BRDF measurements, respectively. In all cases except the Oren–Nayar, the Zernike and the polynomial models, only one specular lobe was used and the diffuse contribution was forced to Lambertian.

The accuracy of the model depends on the degree of the polynomial p . However, increasing accuracy is gained at the expense of introducing additional parameters in the model. To illustrate this situation, we rendered a vase model by employing polynomial approximations to one

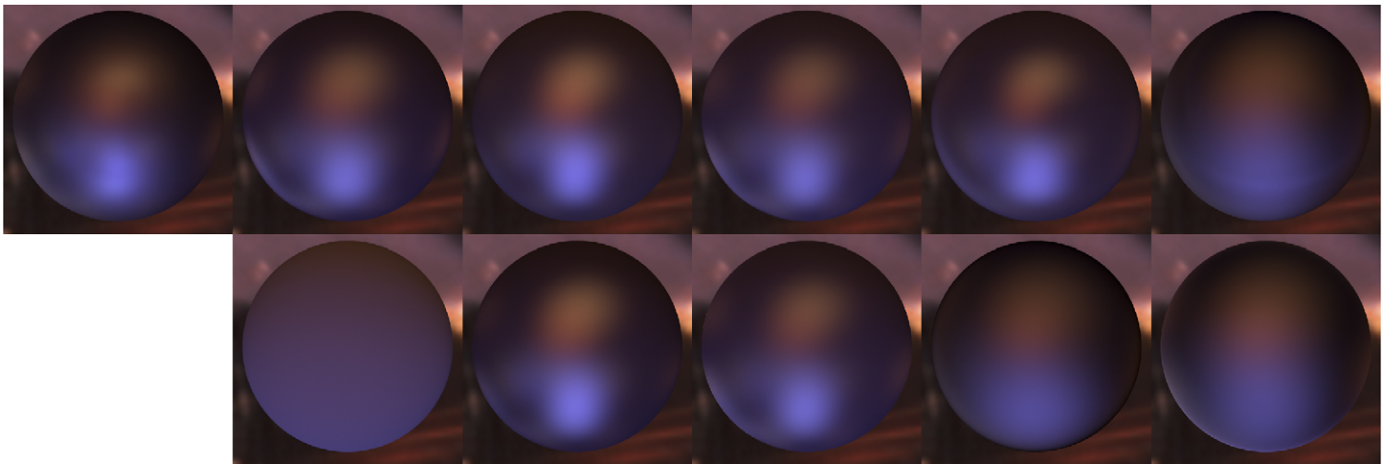


Fig. 5. Renderings of spheres based on the measured “blue-metallic-paint” [2] BRDF with the eight models. From left to right; top row: measured, Ashikhmin–Shirley, Blinn–Phong, Cook–Torrance, Lafortune, Polynomial model (Ashikhmin–Shirley, $p = 3$). Bottom row: Oren–Nayar, Ward, Ward–Duer, Polynomial model (Lafortune, $p = 5$), Polynomial model (Ward, $p = 5$).

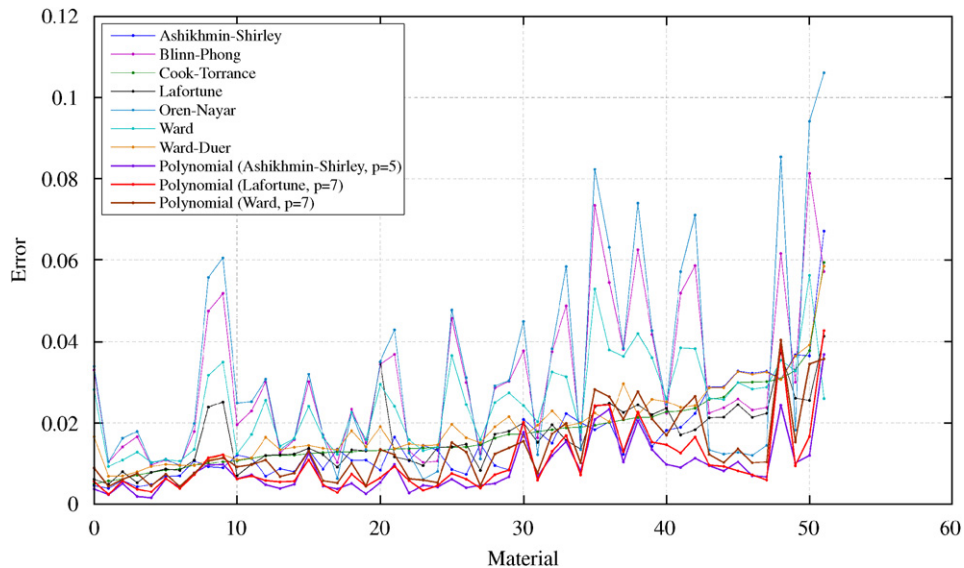


Fig. 6. The L2 error of eight analytic models to CURET data set of 52 isotropic BRDFs. The materials are sorted in the errors of the Cook–Torrance model for visualization purpose.

lobe Lafortune model with a global illumination using Cornell *Cayman lacquer* and *CURET quarry tile*. Fig. 2 shows vase models placed in a lighting environment with polynomial models of varying degrees. The corresponding L2 errors were also computed and given in Table 1. It is seen from the figure and from the L2 errors that the image quality improves with the polynomial models of higher degrees. The difference between the specular reflectance of the materials is clearly observed.

In order to compare the proposed linear model with some of its competitors, we have selected eight commonly used models, namely Ashikhmin–Shirley [28], Blinn–Phong [32], Cook–Torrance [7], Lafortune [3], Oren–Nayar [9], Ward [13], Ward–Duer [33], and Zernike polynomials (of order 8) [19]. Five different diffuse and glossy materials, namely Felt, Orange peel, Quarry tile, Slate(a), Slate(b)

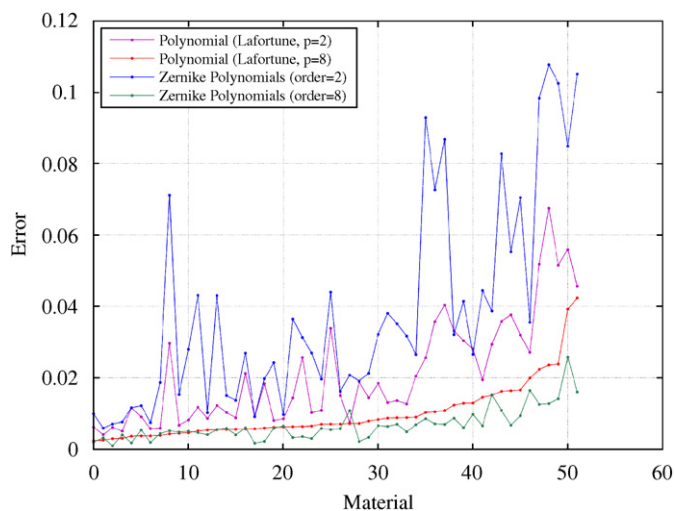


Fig. 7. The fitting errors (L2 errors) of two analytic linear models to CURET data set of 52 isotropic BRDFs. The BRDFs are sorted in the errors of the Polynomial model (Lafortune, $p = 8$) (red) for visualization purpose. For our model, p is both 2 (9 free parameters) and 8 (21 free parameters). For Zernike polynomials, both order 2 (5 free parameters) and order 8 (55 free parameters) are calculated.

from the CURET data set were selected and all models including the polynomial models of various degrees were fitted to these data sets. Fig. 3 illustrates the rendered spheres based on using each of the above models and the materials. Real images were not available for the CURET data set but we consider the images based on Zernike polynomials as close approximations to the real images and used them as test images for comparison purpose. It may be noted from the figure that the images obtained by the proposed model show close agreement with those of Zernike polynomials. For a quantitative comparison L2 metric was used. Table 2 compares the BRDF models on the basis of the L2 for various materials.

Fig. 4 shows Phlegmatic Dragon models obtained using the polynomial models and seven non-linear BRDF models based on “green-latex” data from Matusik et al. [2]. Measured image to represent the reference image was also shown in the figure. Difference images between the reference image and the rendered image were obtained as in Ref. [34] and displayed in the corresponding insets in such a way that the darker portions of the difference image indicate higher difference. To improve visibility all difference images in the figure are standardized using the same scaling factor. For each model, corresponding L2 errors and peak signal-to-noise ratio (PSNR) values based on six isotropic materials including beige-fabric, blue-metallic-paint, dark-blue-paint, green-latex, orange-paint and yellow-plastic were calculated and presented in Table 3. We observe that L2 errors based on our polynomial models are uniformly smaller than the corresponding errors based on all other models including the Ashikhmin–Shirley, the Lafortune and the Ward models.

Rendering of the spheres based on three different polynomial approximations for “blue-metallic-paint” which is known to be highly glossy material is presented in Fig. 5. The corresponding L2 errors and the PSNR values are given in Table 3. Visual inspection of the rendered spheres indicates that the underlying polynomial models did not provide satisfactory approximations for this highly glossy material.



Fig. 8. Left: image based on the measured BRDF “orange-paint” [2]. Middle: scene rendered using polynomial model (Lafortune, $p = 5$). Right: scene rendered using the same polynomial model (Lafortune, $p = 5$) with incoming and outgoing light directions exchanged. Insets show a difference between the reference and the rendered images.

Table 4
Average fitting times (in seconds) of BRDF models on CURET dataset (52 isotropic materials were chosen)

BRDF model	Average fitting time	BRDF model	Average fitting time
Ashikhmin–Shirley	0.33175	Ward	0.29025
Blinn–Phong	0.38900	Ward–Duer	0.26925
Cook–Torrance	0.34725	Poly. (Ashikhmin–Shirley, $p = 5$)	0.01567
Lafortune	0.36540	Poly. (Lafortune, $p = 7$)	0.01565
Oren–Nayar	0.15596	Poly. (Ward, $p = 7$)	0.01565

Times are calculated on a Pentium D 2.66 GHz 2 Gb RAM machine.

To provide a further comparison with the other BRDF models, we fitted each of the previously used seven BRDF models and the polynomial models to the 52 different isotropic materials from the CURET data set. L2 errors based on the measured BRDF were plotted in Fig. 6. In this figure, the materials are sorted with respect to L2 errors of the Cook–Torrance model to provide a better visualization. It can be seen from the figure that our polynomial approximations based on the Ashikhmin–Shirley, the Lafortune and the Ward models have the lowest errors in 45, 40 and 23 cases out of 52 cases, respectively, as compared to the non-linear models listed as the first seven rows of the legend of Figs. 6 and 7 compares the polynomial models with the Zernike Polynomial models in a similar fashion. It is seen that representational adequacy of the models increases with the degree of the models. However, the numbers of the parameters estimated are 21 and 55 for the polynomial approximation based on Lafortune model of degree 8 and the Zernike model of order 8, respectively.

The reciprocity property of the polynomial model for the Lafortune model is clearly seen in Fig. 8.

Average fitting times in seconds of fitting above BRDF models to the 52 isotropic materials in the CURET data set were obtained on a Pentium D 2.66 GHz Computer are shown in Table 4. The polynomial models have resulted the lowest running times overall of the other models.

7. Conclusions

In this paper, we have presented a general approach for modeling both isotropic and anisotropic diffuse and glossy materials. The proposed model for describing surface reflectance is a response surface model based on principal component transformations of the underlying explanatory variables. Since our model essentially is a multivariable polynomial, it is flexible and can be easily fitted to experimental data by employing standard multiple regression techniques. Furthermore, the underlying model is linear in the parameters and therefore no difficulty is encountered in parameter estimation as opposed to BRDF models having non-linear parameters.

Using several BRDF test data, the corresponding renderings with the proposed model were shown to be

visually convincing and can be used for real-time rendering.

In this paper, our approach was demonstrated using the Ashikhmin–Shirley, the Lafortune and the Ward representations. However, the proposed approach is not restricted to these cases only and can be generalized to approximate other models as well.

An interesting property of our approach in modeling BRDF is that the variables used in the polynomial model are uncorrelated so that under normality assumption they can be treated as independent variables. Although normality assumption does not always hold in most practical applications, this property of the uncorrelated variables could be exploited to provide a ground for developing an efficient importance sampling procedure. Our work will be continued along the line of approximating some other non-linear models by the polynomial models and developing corresponding importance sampling schemes.

With the introduction of response surface models for representing BRDFs we believe that the corresponding rendering process can be easily implemented for a wide range of applications and will be of help for the researchers who seek developing general, flexible and computationally efficient models.

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