

# A Genetic Algorithm Based Heterogeneous Subsurface Scattering Representation

**Murat Kurt<sup>1</sup>**

**<sup>1</sup>International Computer Institute, Ege University**

In this paper, we present a novel heterogeneous subsurface scattering representation, which is based on a combination of Singular Value Decomposition and genetic optimization techniques.

# Introduction



Jensen et al. [JMLH01]  
*Standard Dipole Model*

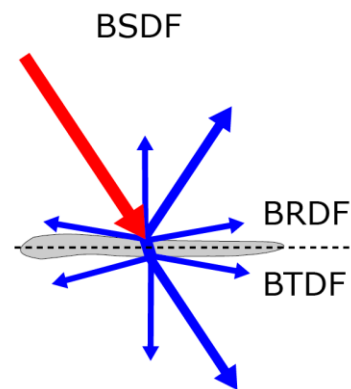
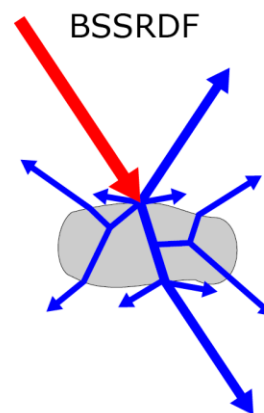
d'Eon and Irving [dI11]  
*Quantized Diffusion Model*

Frisvad et al. [FHK14]  
*Directional Dipole Model*

*Unbiased Path Tracing  
Reference (Marble Material)*



Jimenez et al. [JZ]\*15]



The most of the homogeneous subsurface scattering models used in the computer graphics community are derived based on this approximation is used to represent homogeneous materials [JMLH01, JB02, FHK14, JZJ\*15].

Nicodemus et al. [NRH\*77] and Frisvad et al. [FJM\*20]

# Introduction



Peers et al. [PvBM\*06]

*A factored heterogeneous model*



Song et al. [STPP09]

*SubEdit: A heterogeneous model*



Kurt et al. [KÖP13]

*A Tucker-based heterogeneous model*

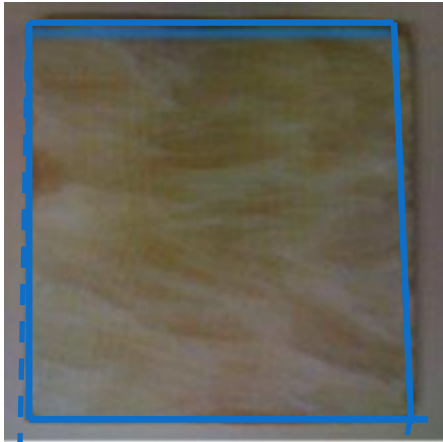


Yatagawa et al. [YTYM20]

*A compression based heterogeneous model*

Therefore, a number of data-driven subsurface scattering representations [PvBM\*06, STPP09, KÖP13, YTYM20] have been proposed. Problem: Accurate and Compact Representation of heterogeneous subsurface scattering. But, it's still an investigation issue to represent measured heterogeneous subsurface scattering data compactly and physically accurately.

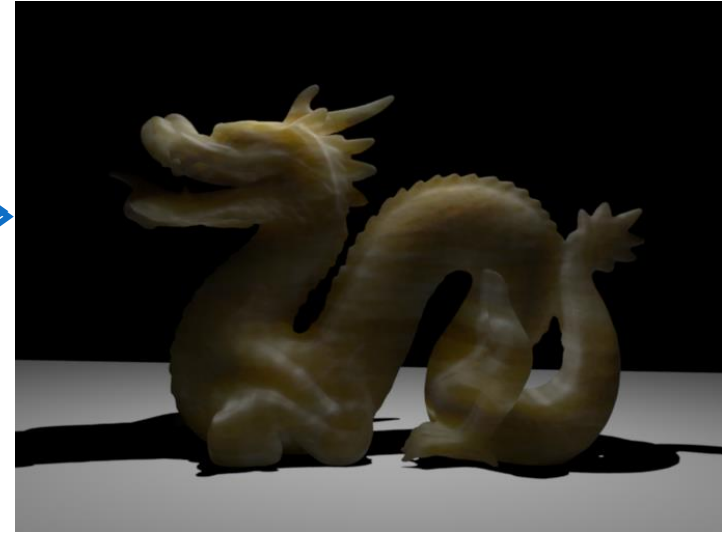
# Goal



Therefore, we focus in this work on the compact and accurate representation of this spatial component of heterogeneous subsurface scattering.



# Goal

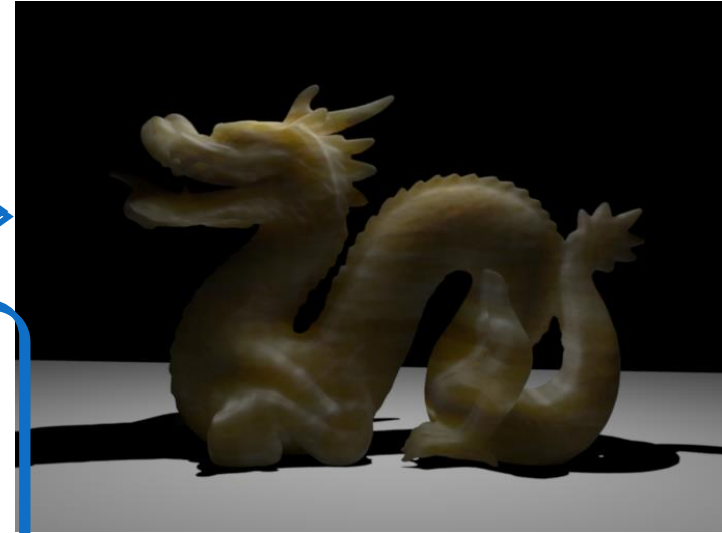


# Goal



Impractical

*Resolution:*  
 $260^2 \times 260^2$   
 $\sim 40\text{GB}$



We will use a combination of Genetic Algorithm (GA) [Mit96] and Singular Value Decomposition (SVD) [PSR13] techniques for this.

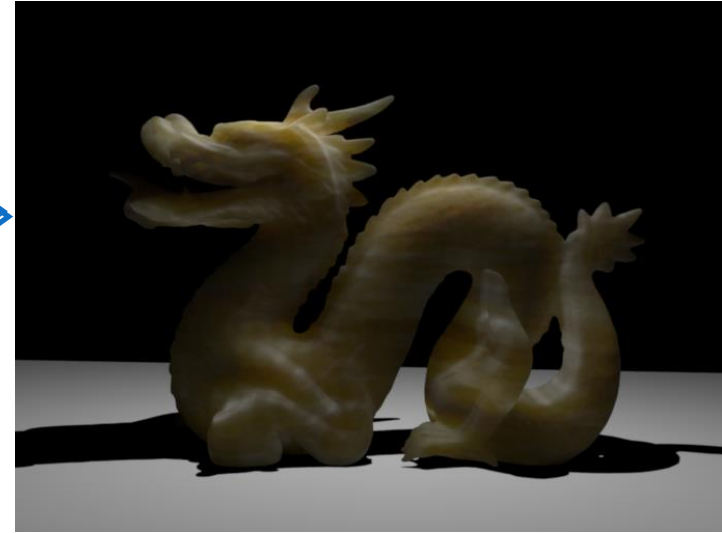




# Goal



Impractical

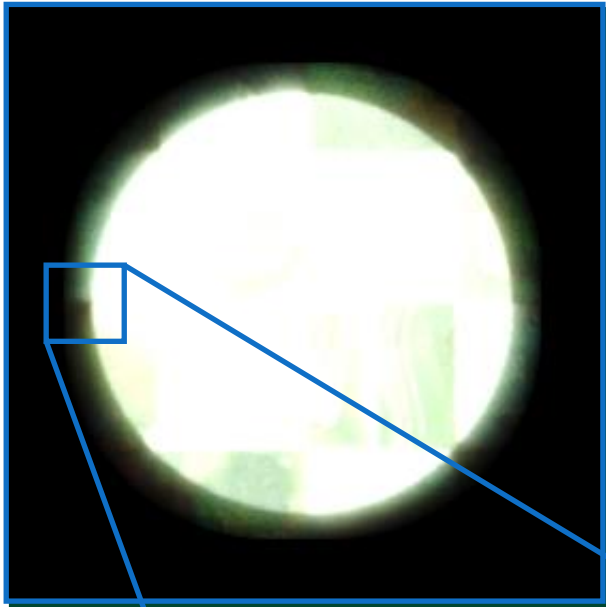


**Compact and Accurate Representation**  
**(Factorization and Genetic Optimization)**

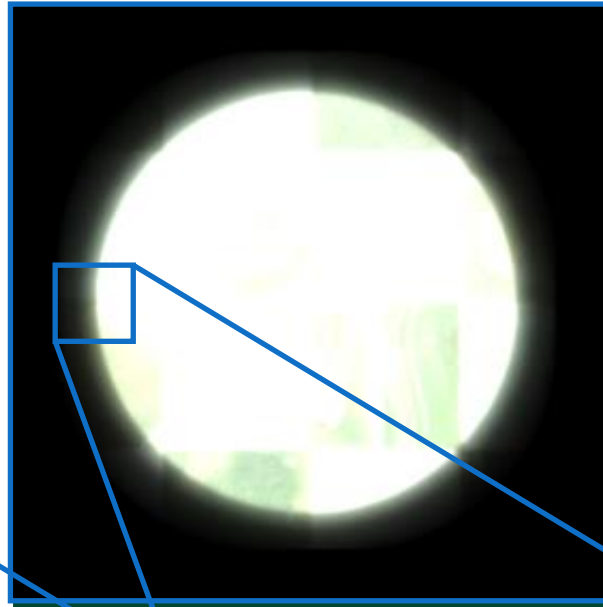
In order to create a good representation, we need to study the structure of the heterogeneous subsurface scattering.

# Heterogeneities: Effect

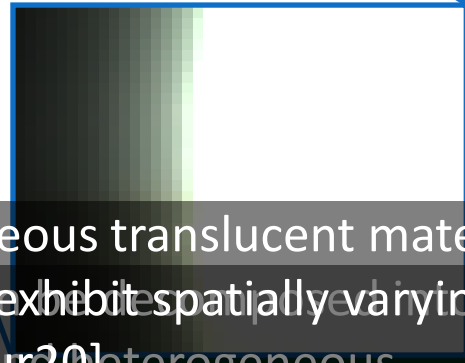
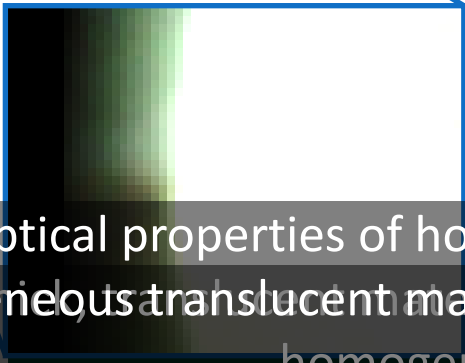
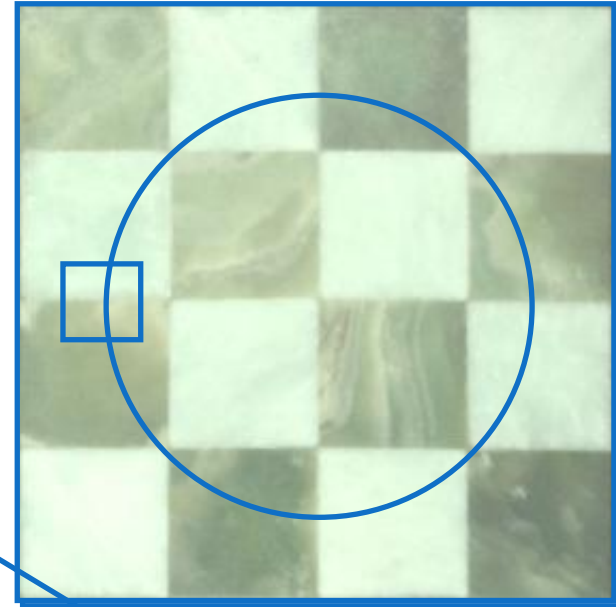
Heterogeneous  
Subsurface Scattering



Homogeneous  
Subsurface Scattering  
(Modulation Texture)



Diffuse Albedo

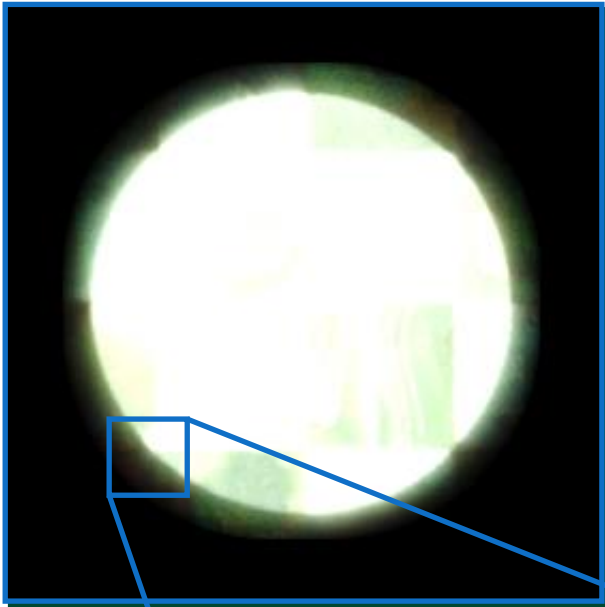


While optical properties of homogeneous translucent materials are constant, heterogeneous translucent materials exhibit spatially varying optical behaviors. [Kur20]



# Heterogeneities: Effect

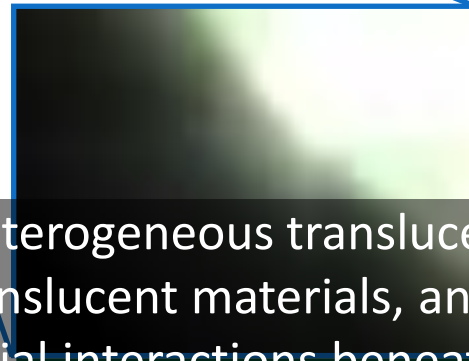
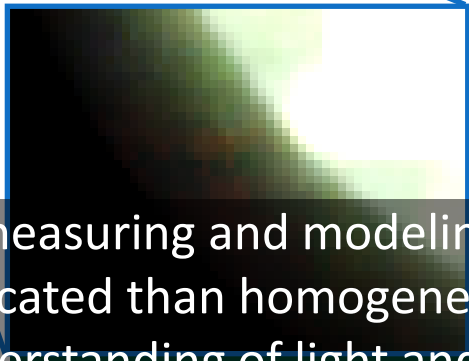
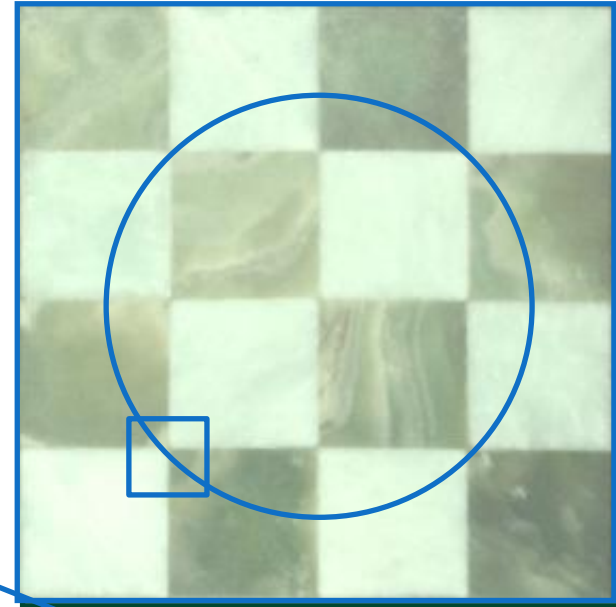
Heterogeneous  
Subsurface Scattering



Homogeneous  
Subsurface Scattering  
(Modulation Texture)

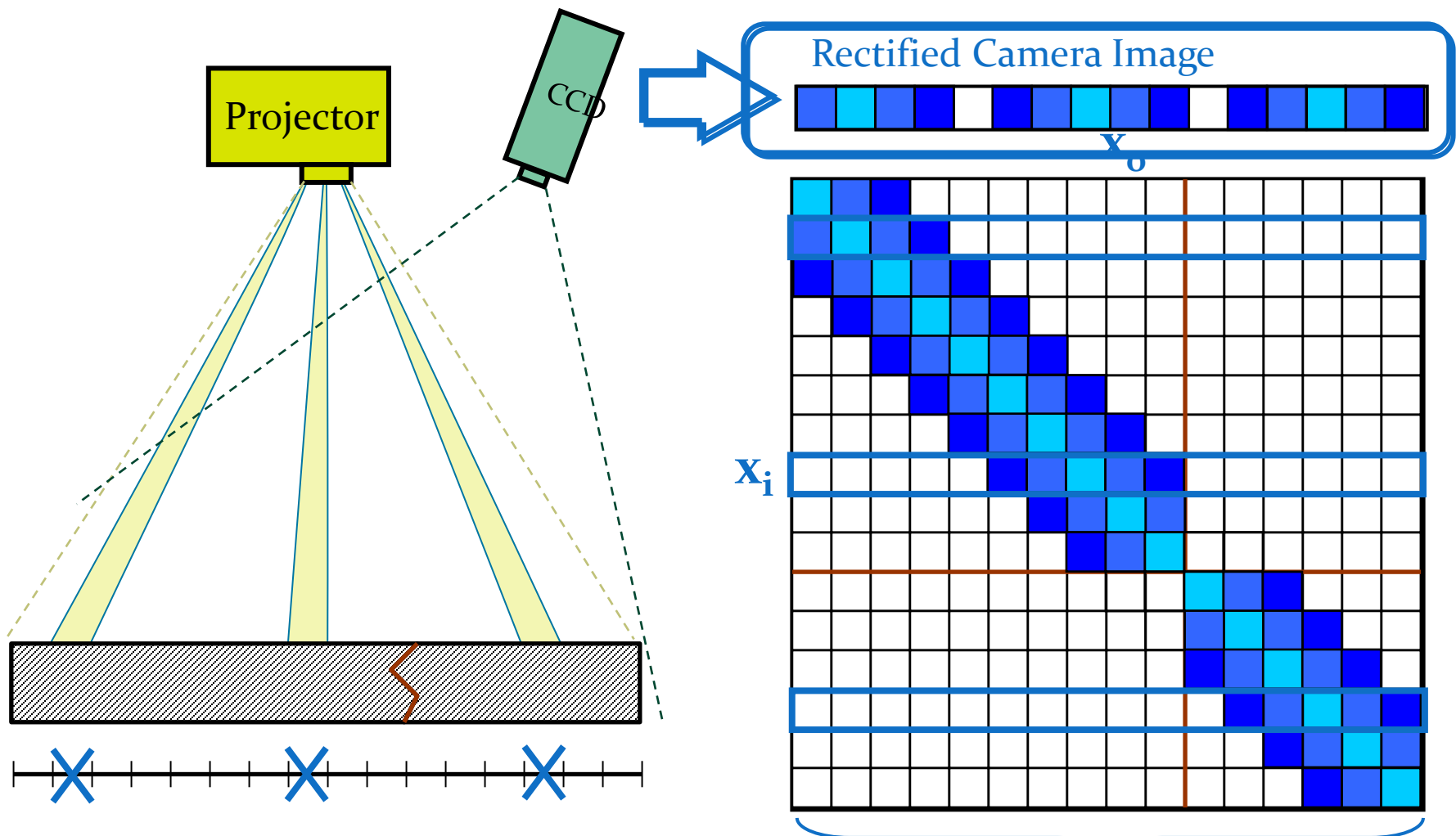


Diffuse Albedo



Therefore, measuring and modeling of heterogeneous translucent materials are much more complicated than homogeneous translucent materials, and requires much better understanding of light and material interactions beneath the surface.

# Acquisition: Multiple Beams (1)



In this work, we used Peers et al.'s [2006] Song et al.'s [2009] heterogeneous  
After these operations, we get 2D subsurface scattering matrix and this matrix can be  
two steps, used in the rendering algorithms.

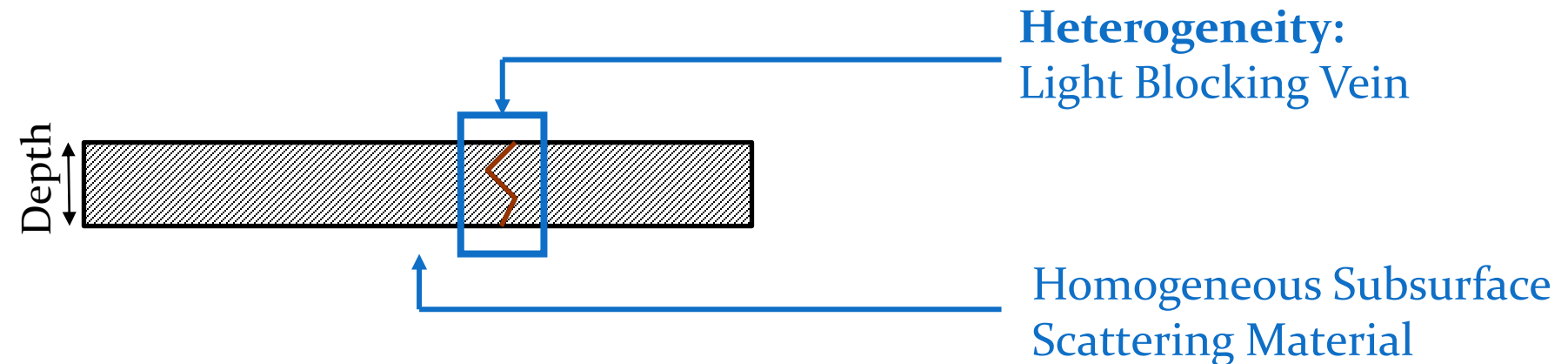
# Illustrative Example (1D)



**Homogeneous Subsurface  
Scattering Material**

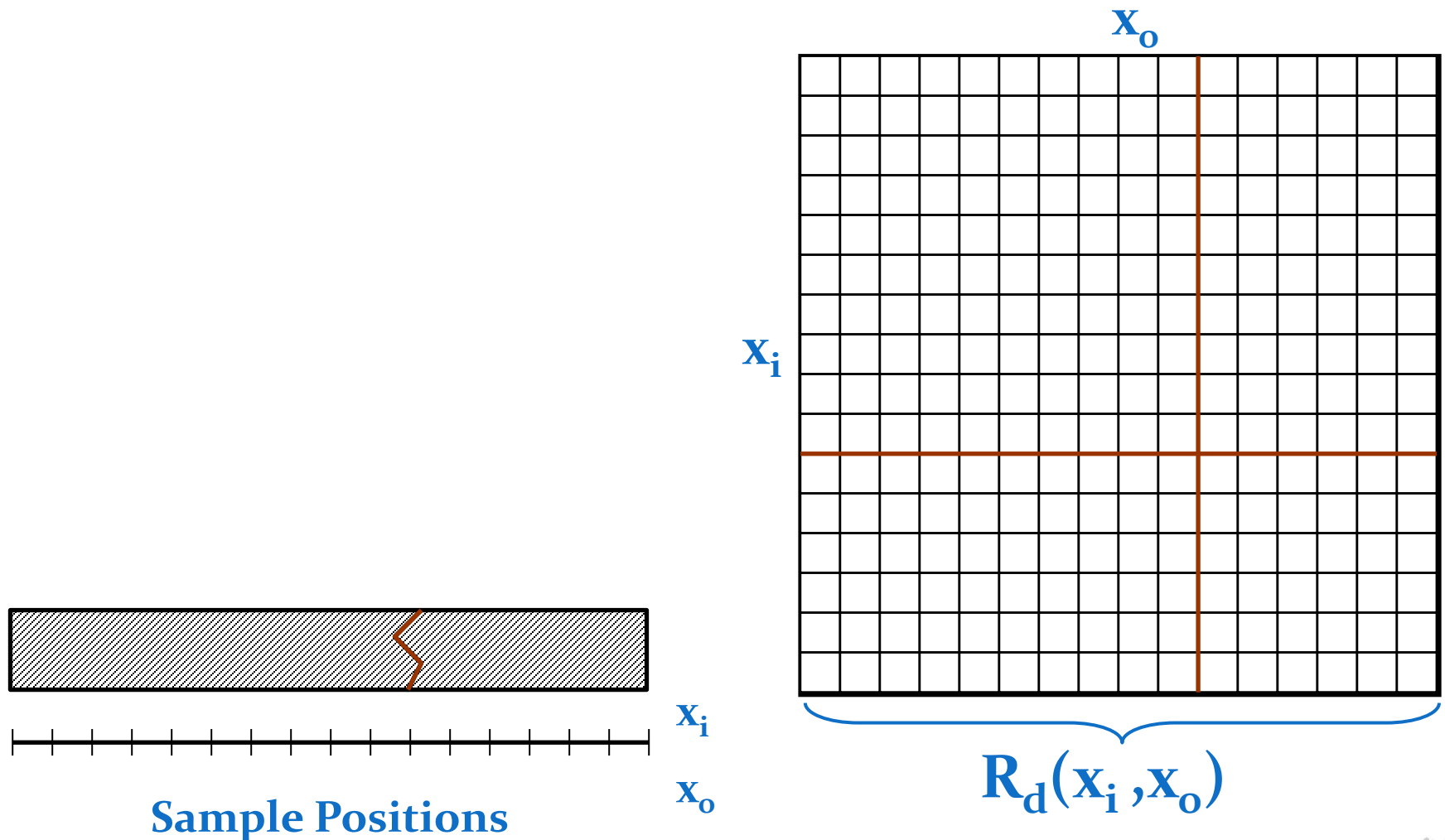
To show how we prepare our subsurface scattering matrix, let's consider the following 1D illustrative example: a planar sample, consisting of a single material exhibiting homogeneous subsurface scattering.

# Illustrative Example (1D)



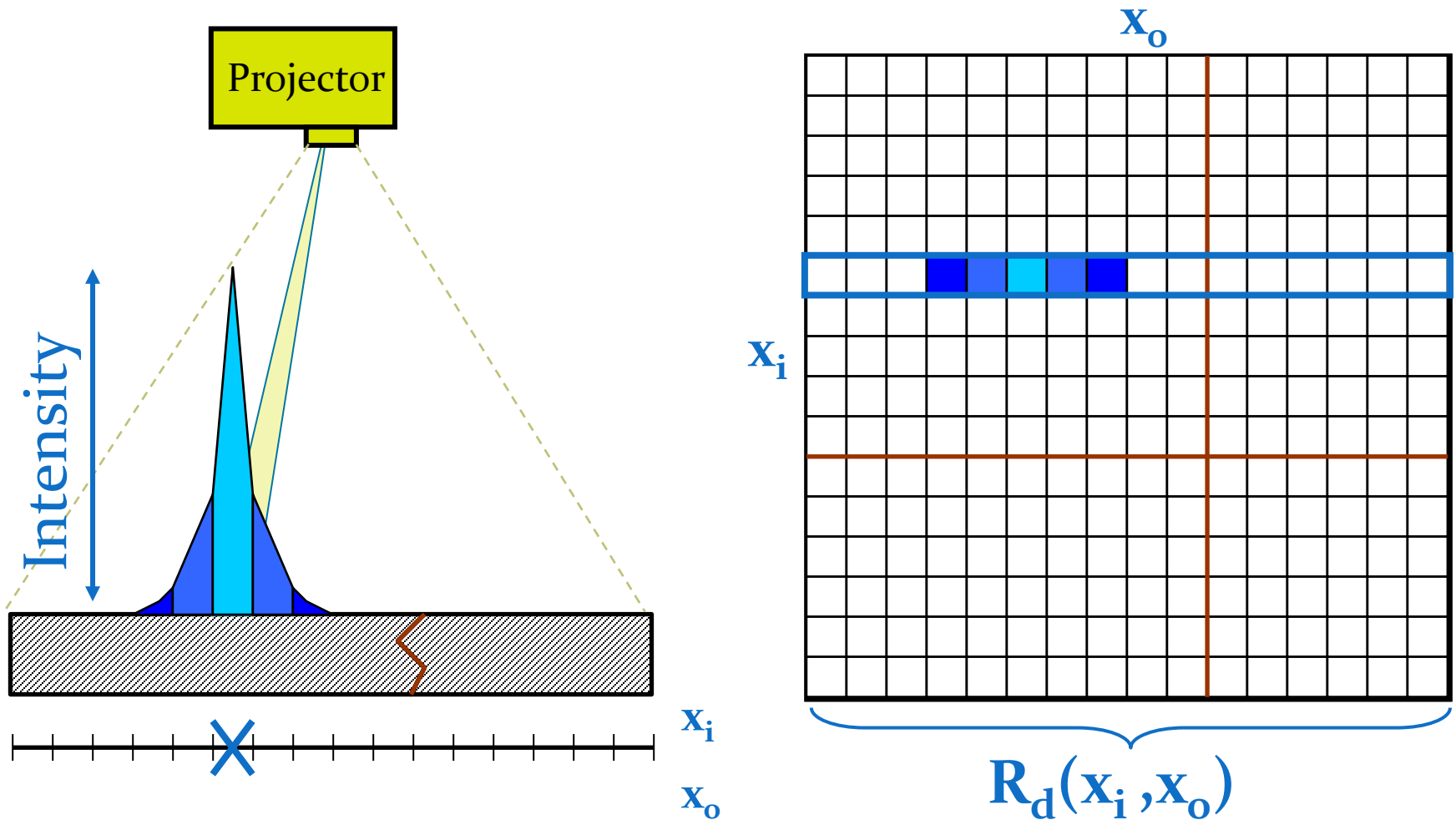
This sample contains a single heterogeneity: a light blocking vein, that blocks a light going from the left side of the sample to the right side, and vice versa.

# Subsurface Scattering Matrix



For simplicity we assume that  $x_i$  and  $x_o$  are parameterized identically over the parameterized sample surface and that both have the same resolution.

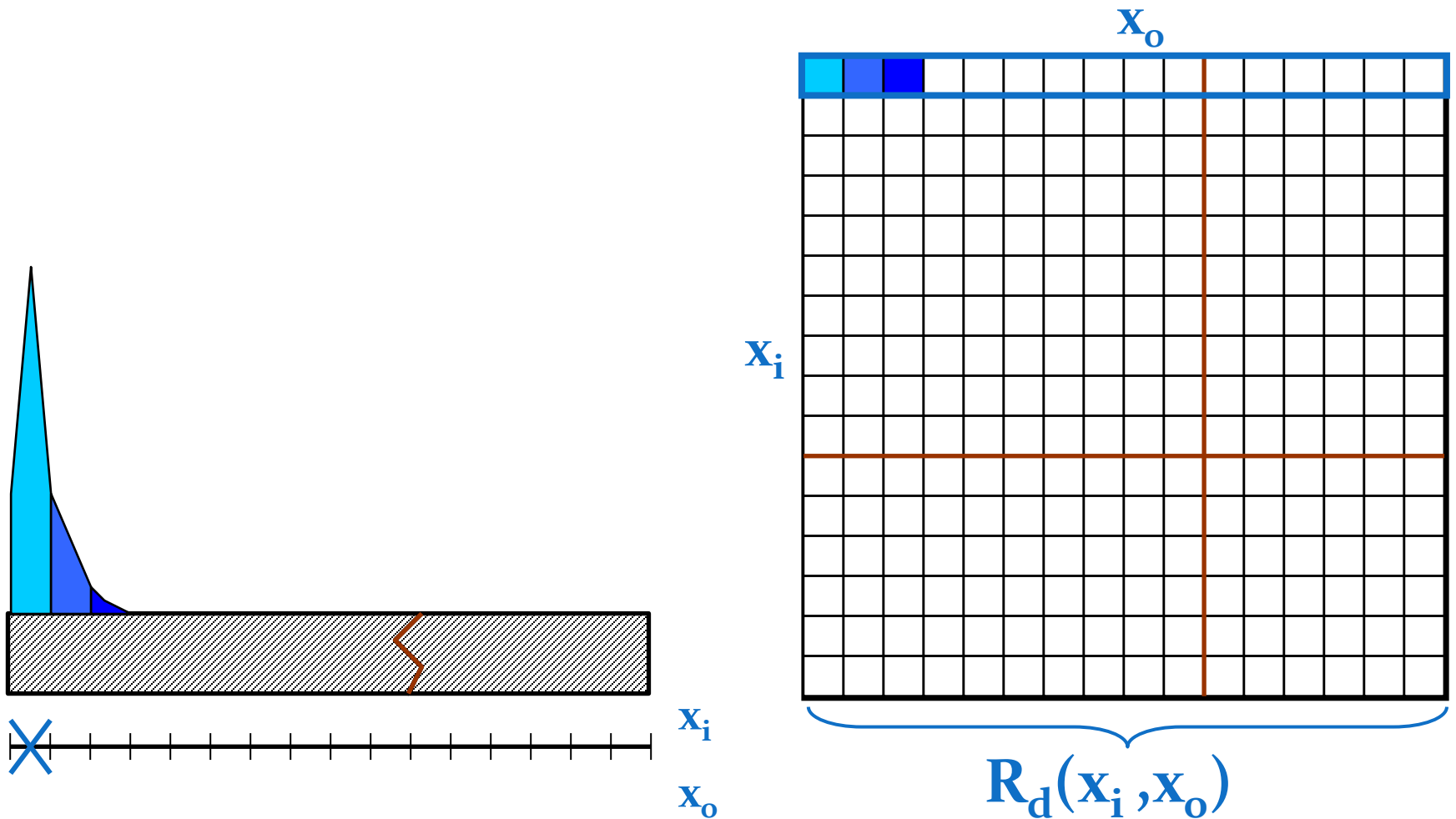
# Subsurface Scattering Matrix



In our case, a projector was used to illuminate surface points, and the observed camera image is copied into the corresponding row of  $R_d$ .



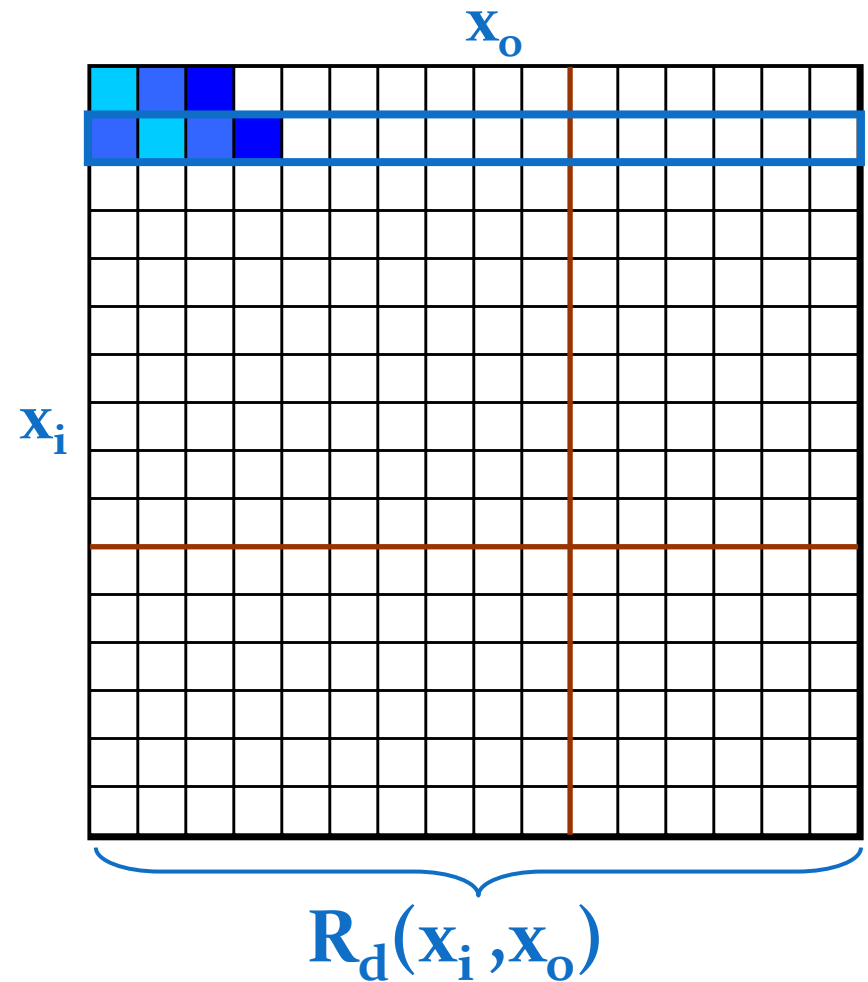
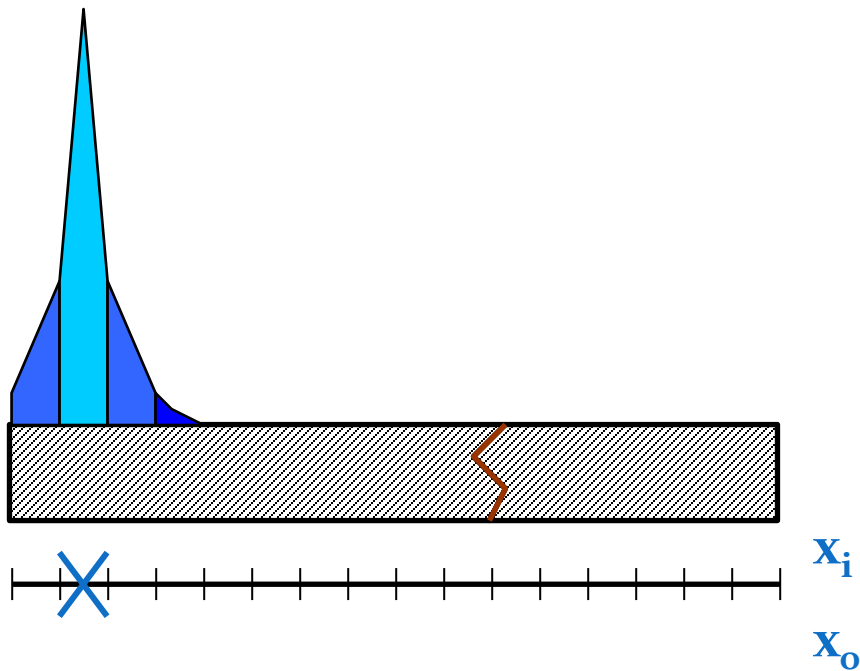
# Subsurface Scattering Matrix



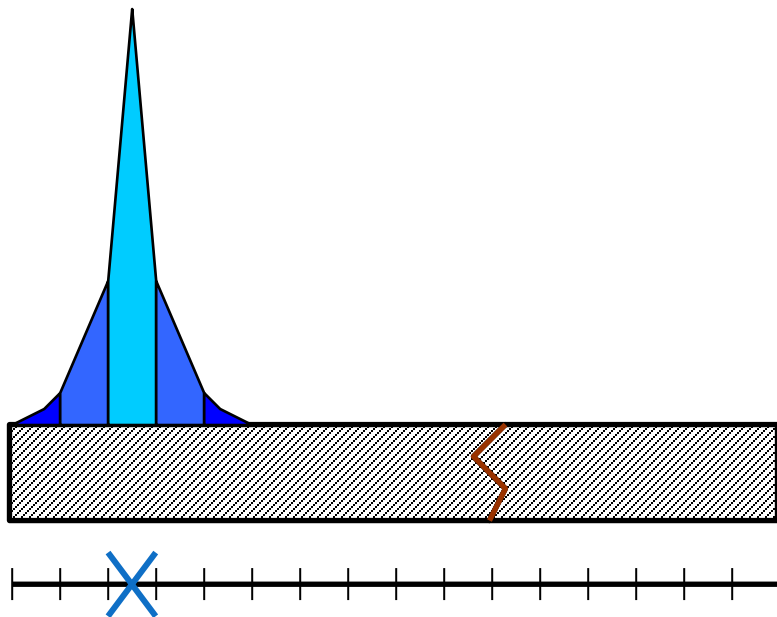
We can repeat this for every incident position and row.



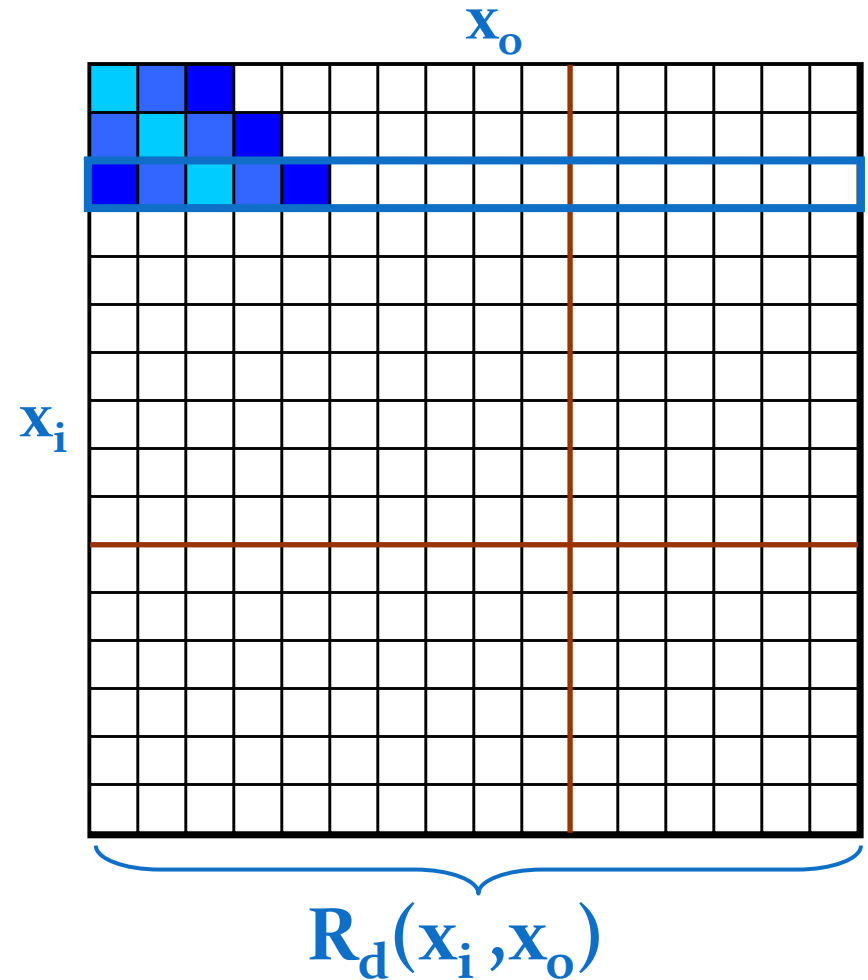
# Subsurface Scattering Matrix



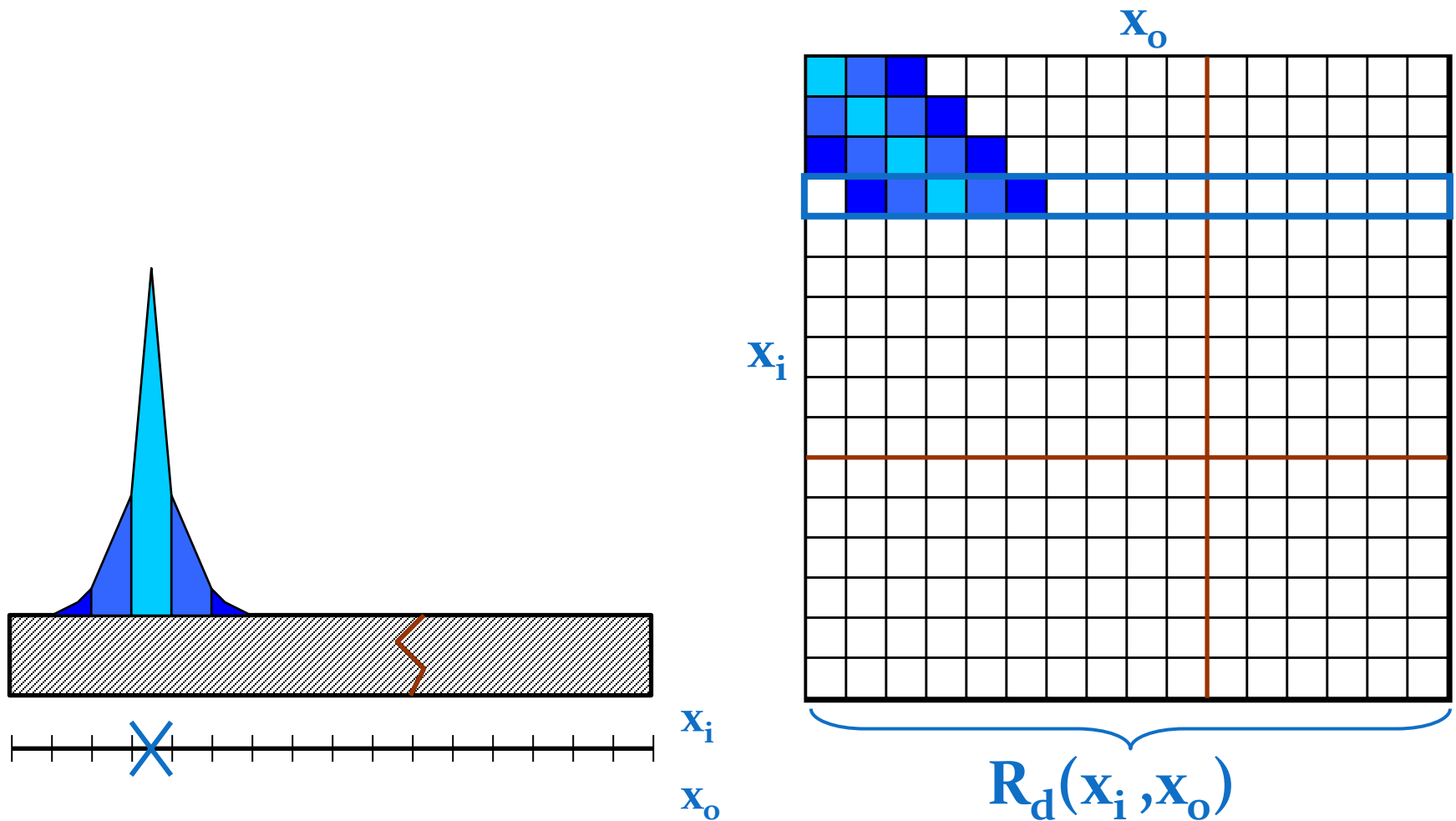
# Subsurface Scattering Matrix



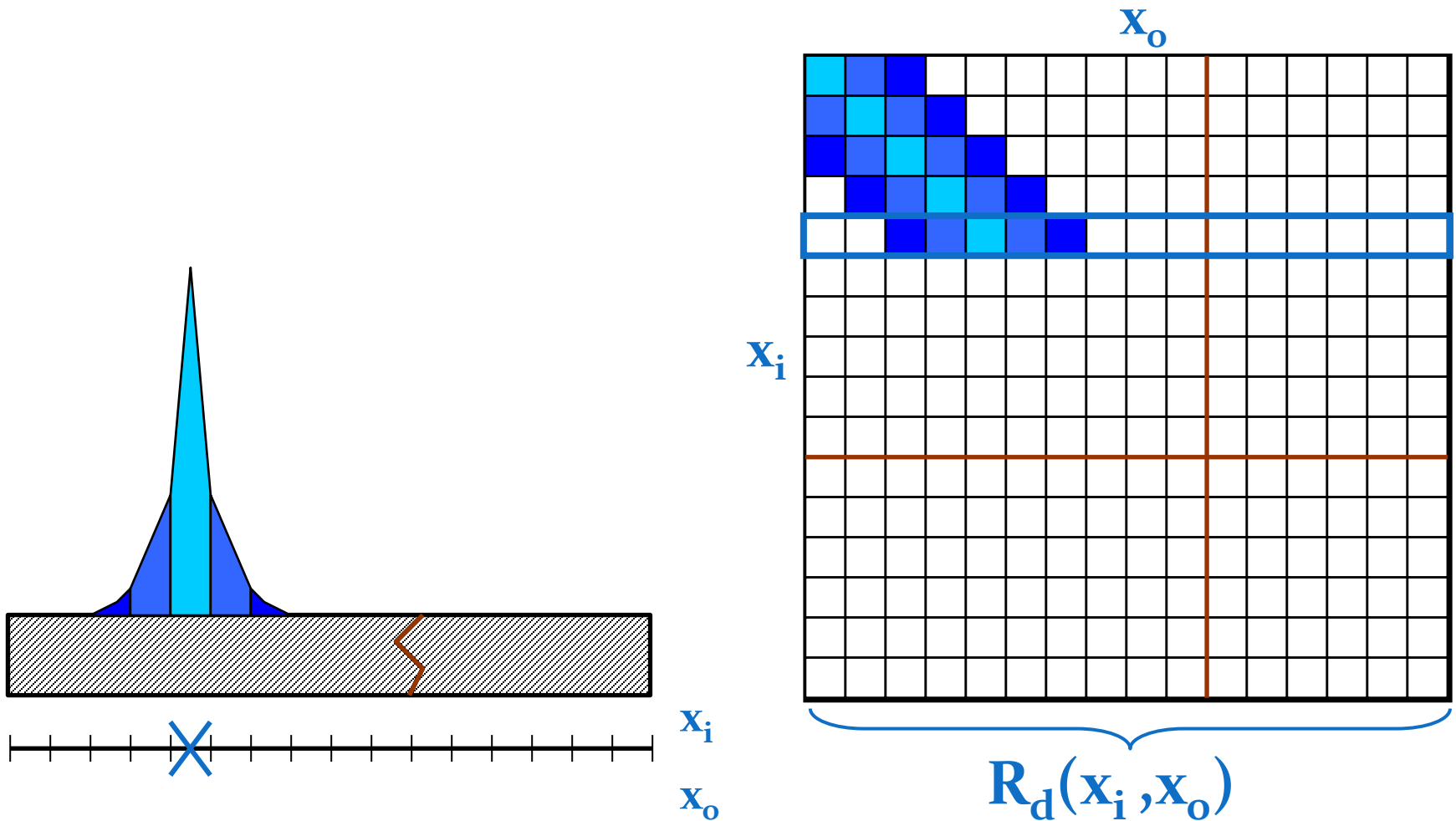
$x_i$   
 $x_o$



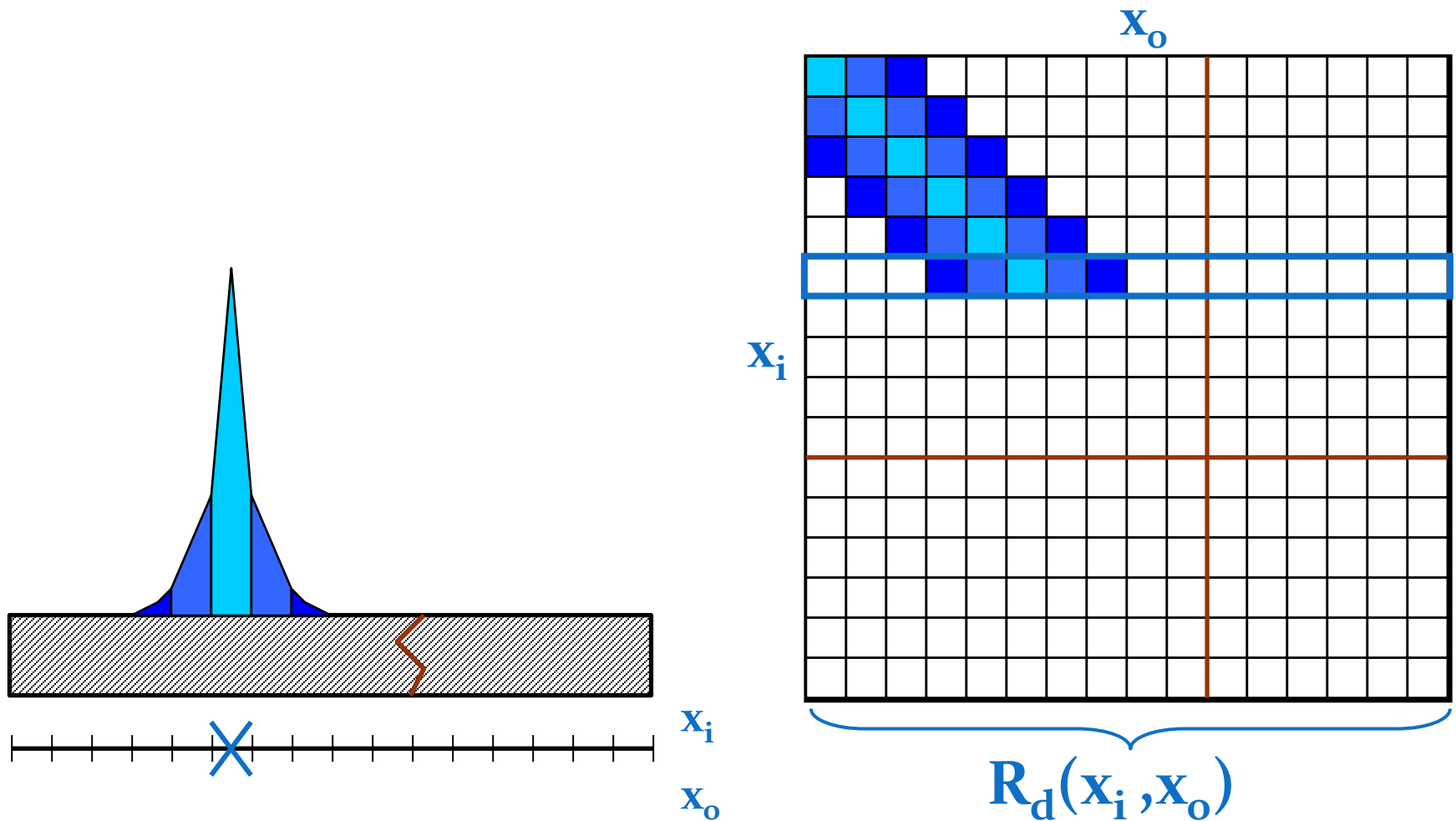
# Subsurface Scattering Matrix



# Subsurface Scattering Matrix

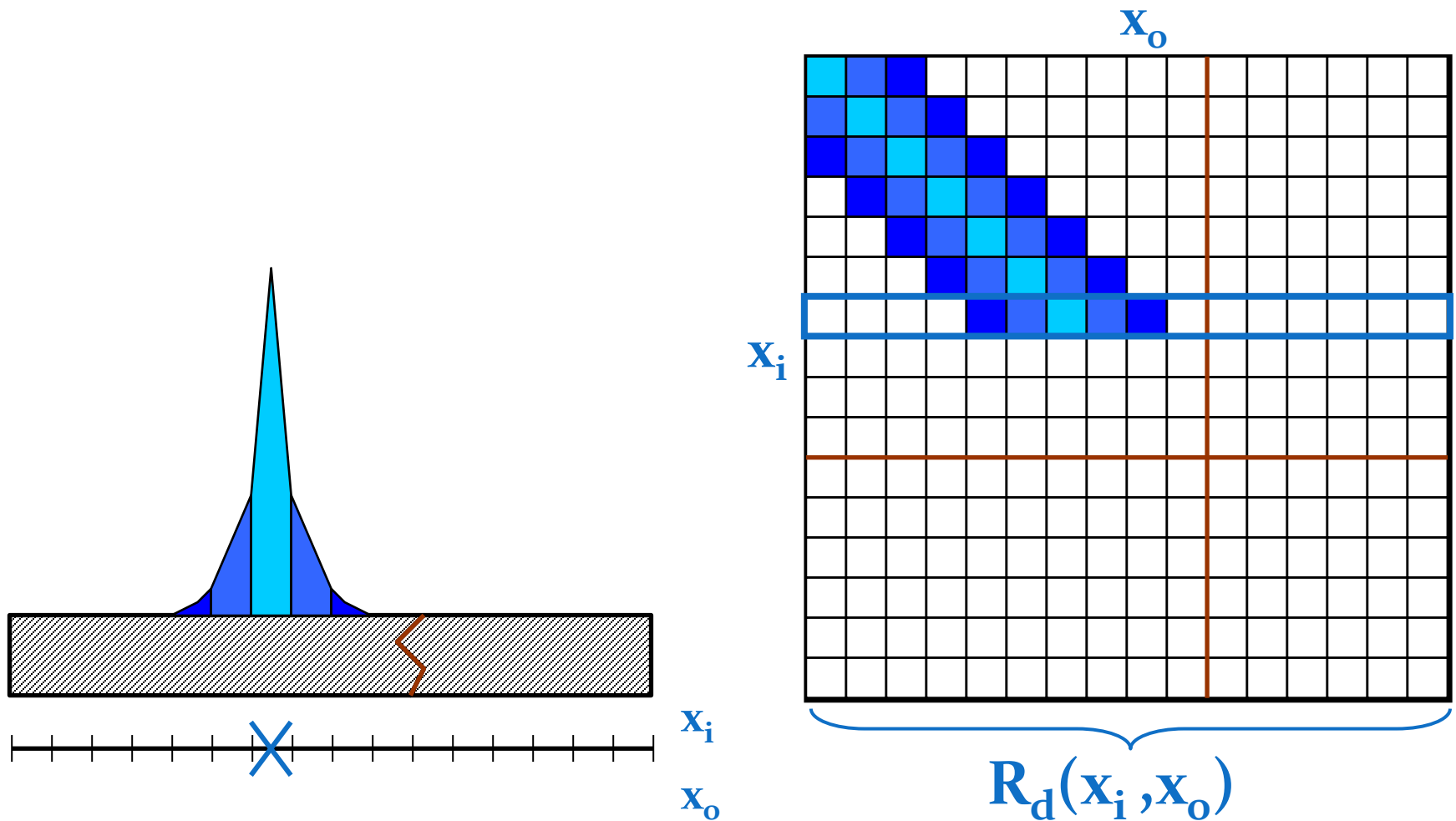


# Subsurface Scattering Matrix

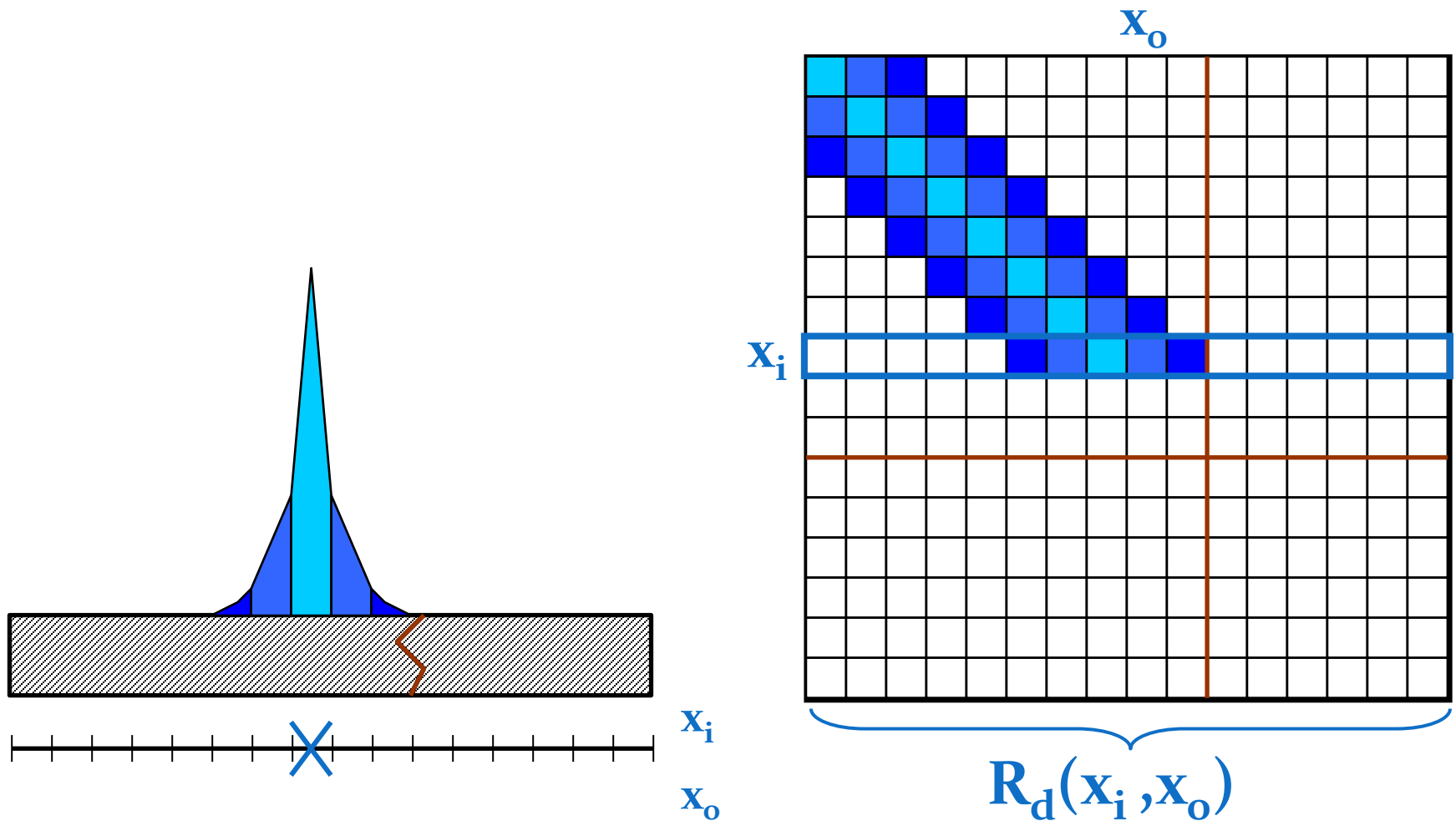




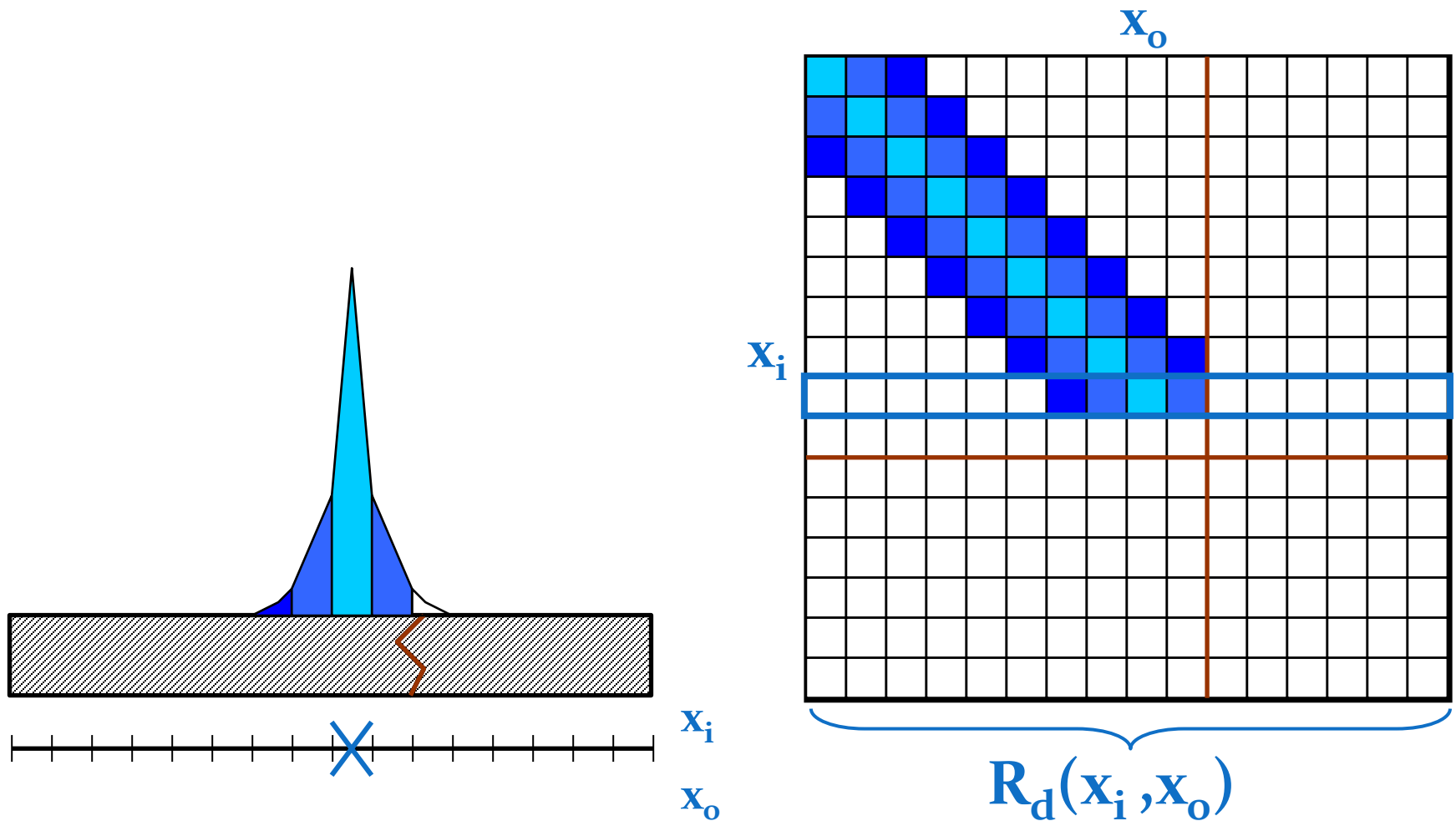
# Subsurface Scattering Matrix



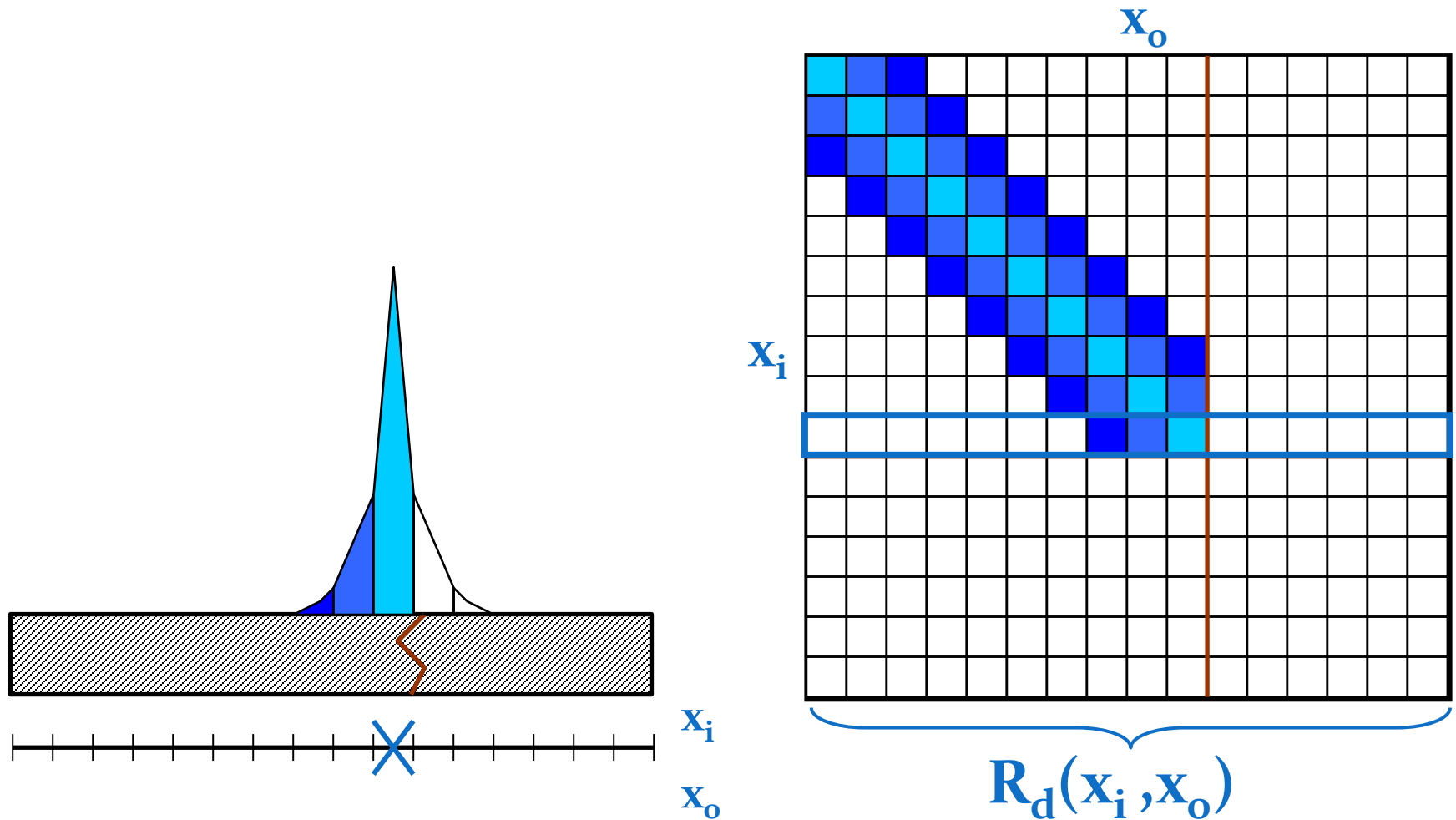
# Subsurface Scattering Matrix



# Subsurface Scattering Matrix

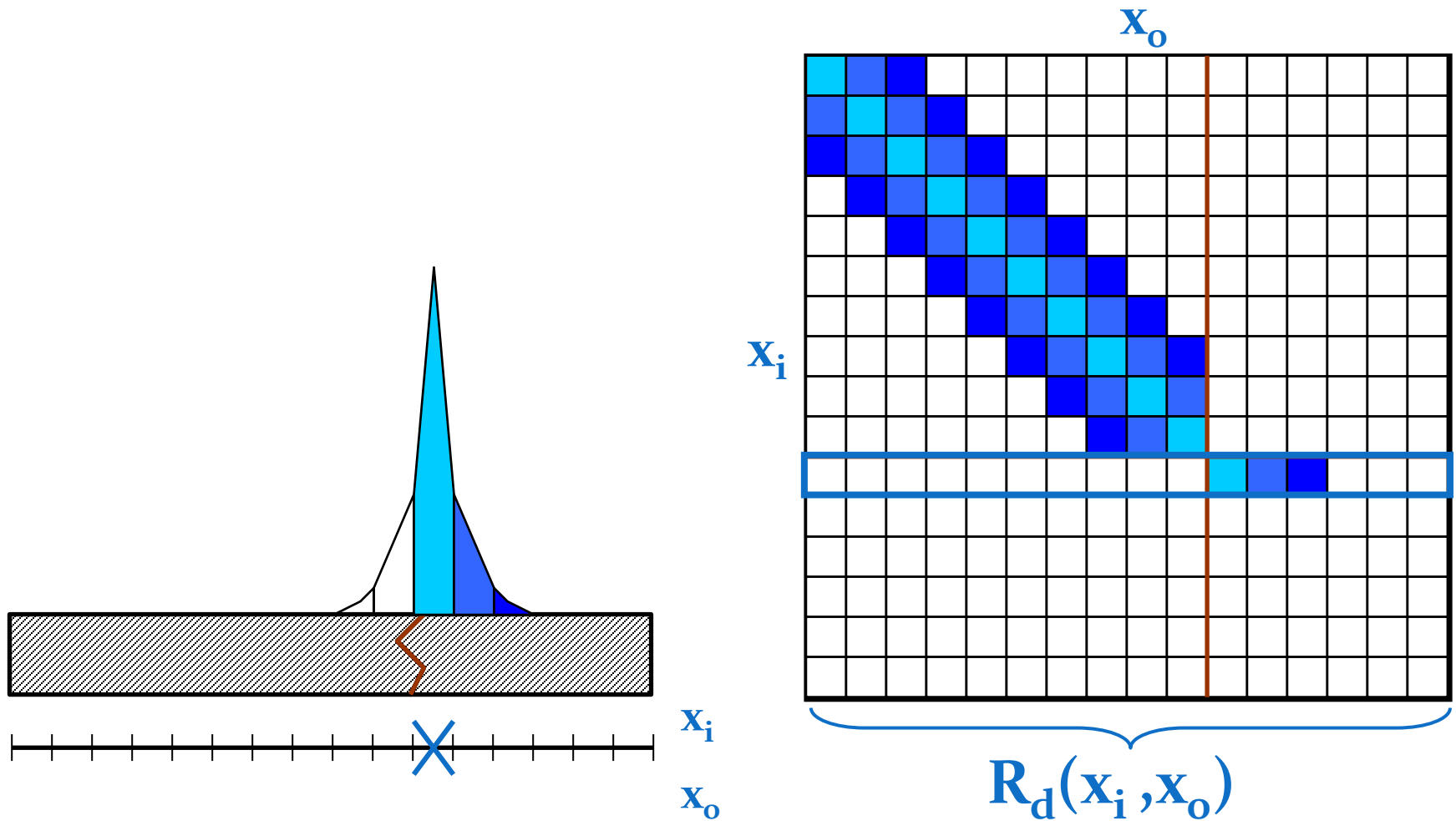


# Subsurface Scattering Matrix

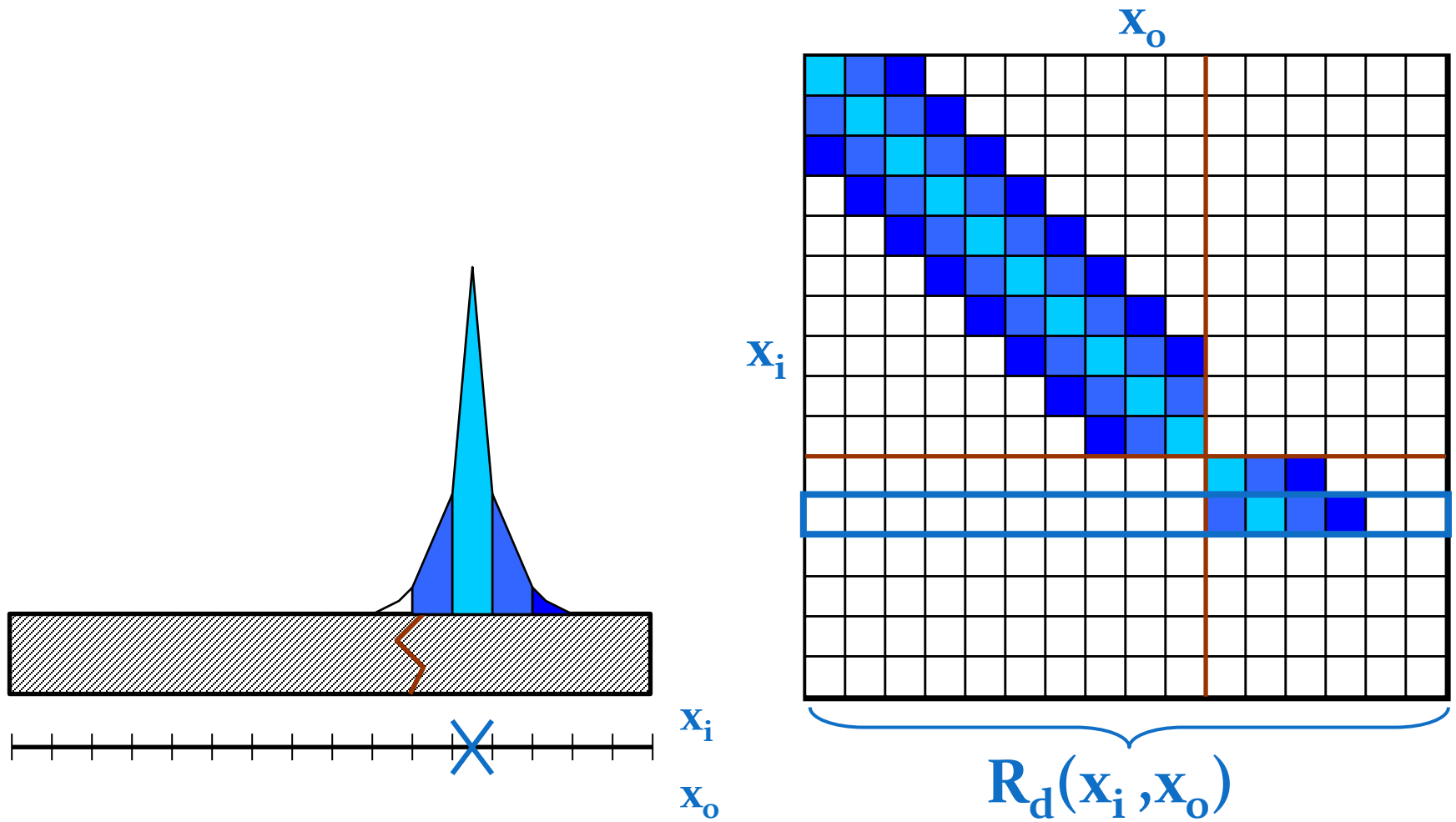


Note that when a point close to the light blocking vein is illuminated, that a part of the homogeneous response is cut off.

# Subsurface Scattering Matrix

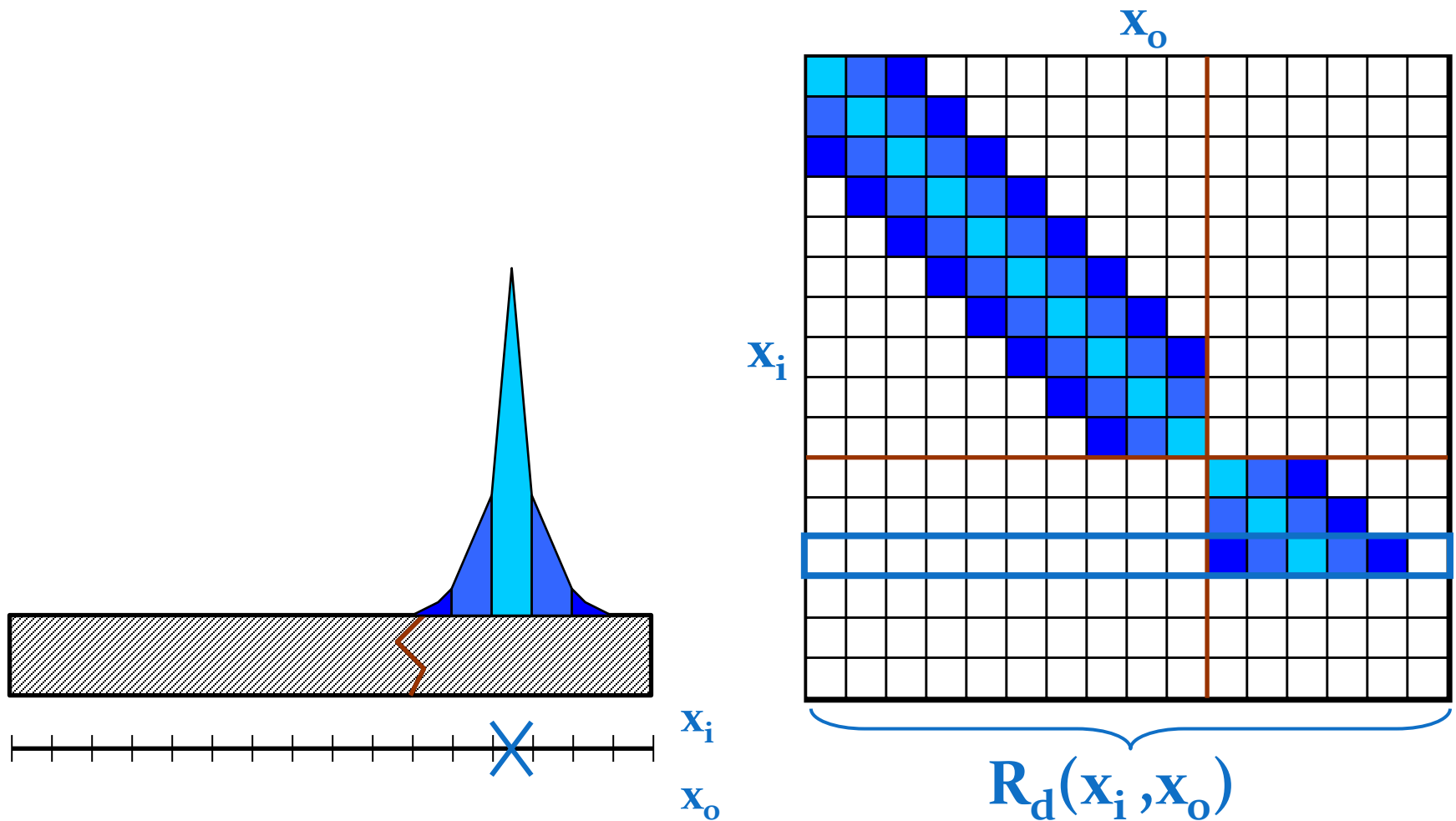


# Subsurface Scattering Matrix

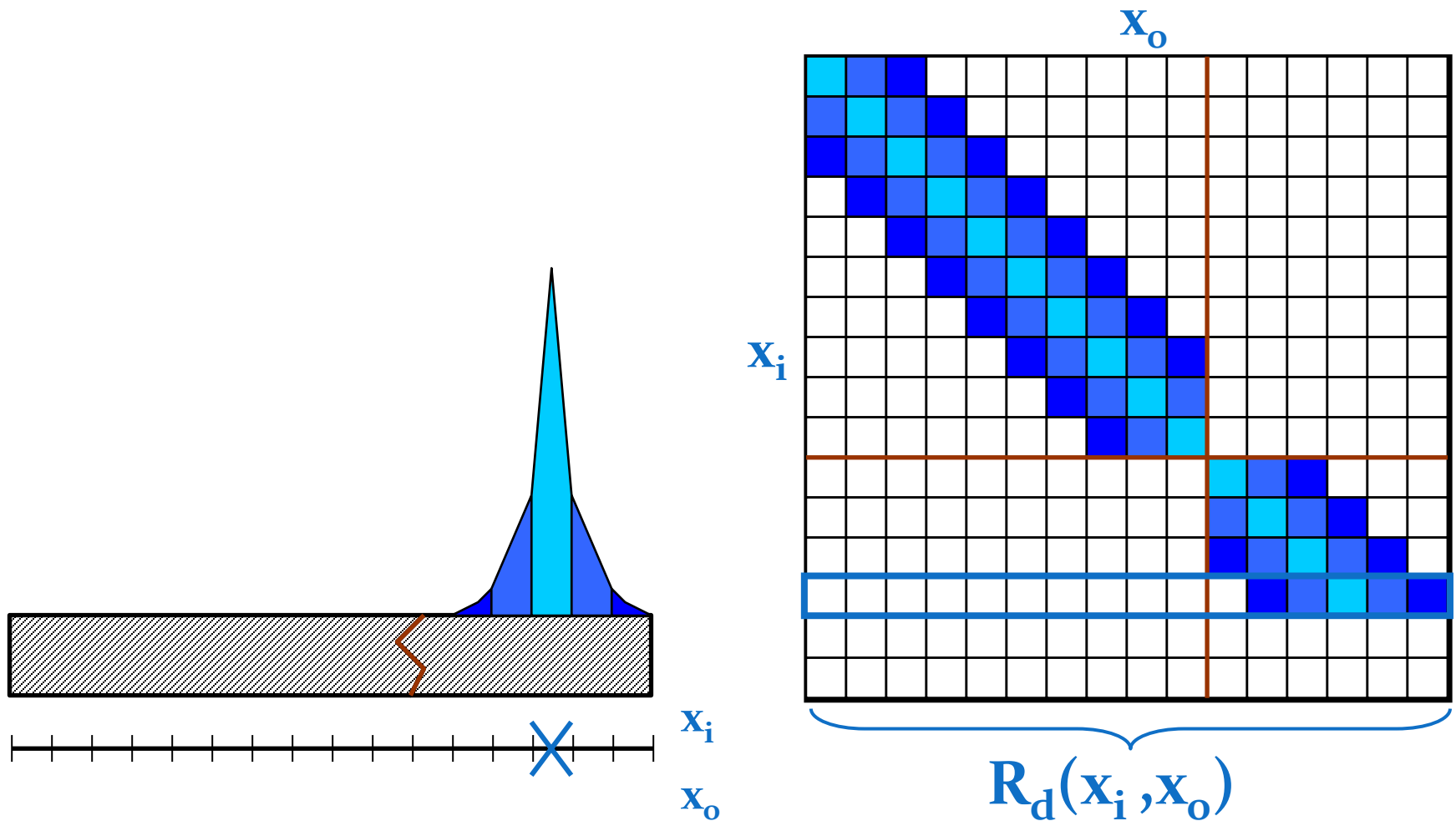




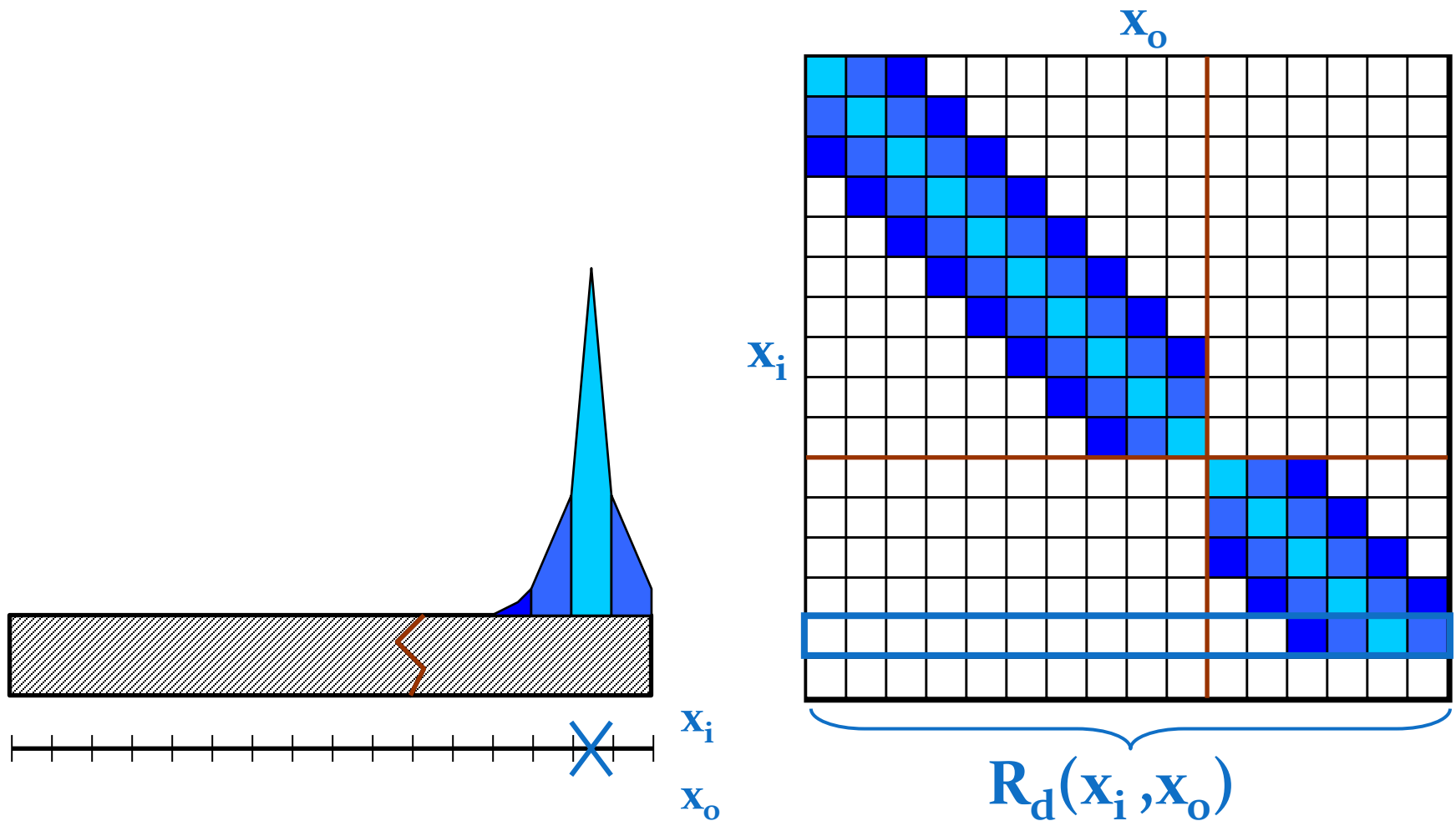
# Subsurface Scattering Matrix



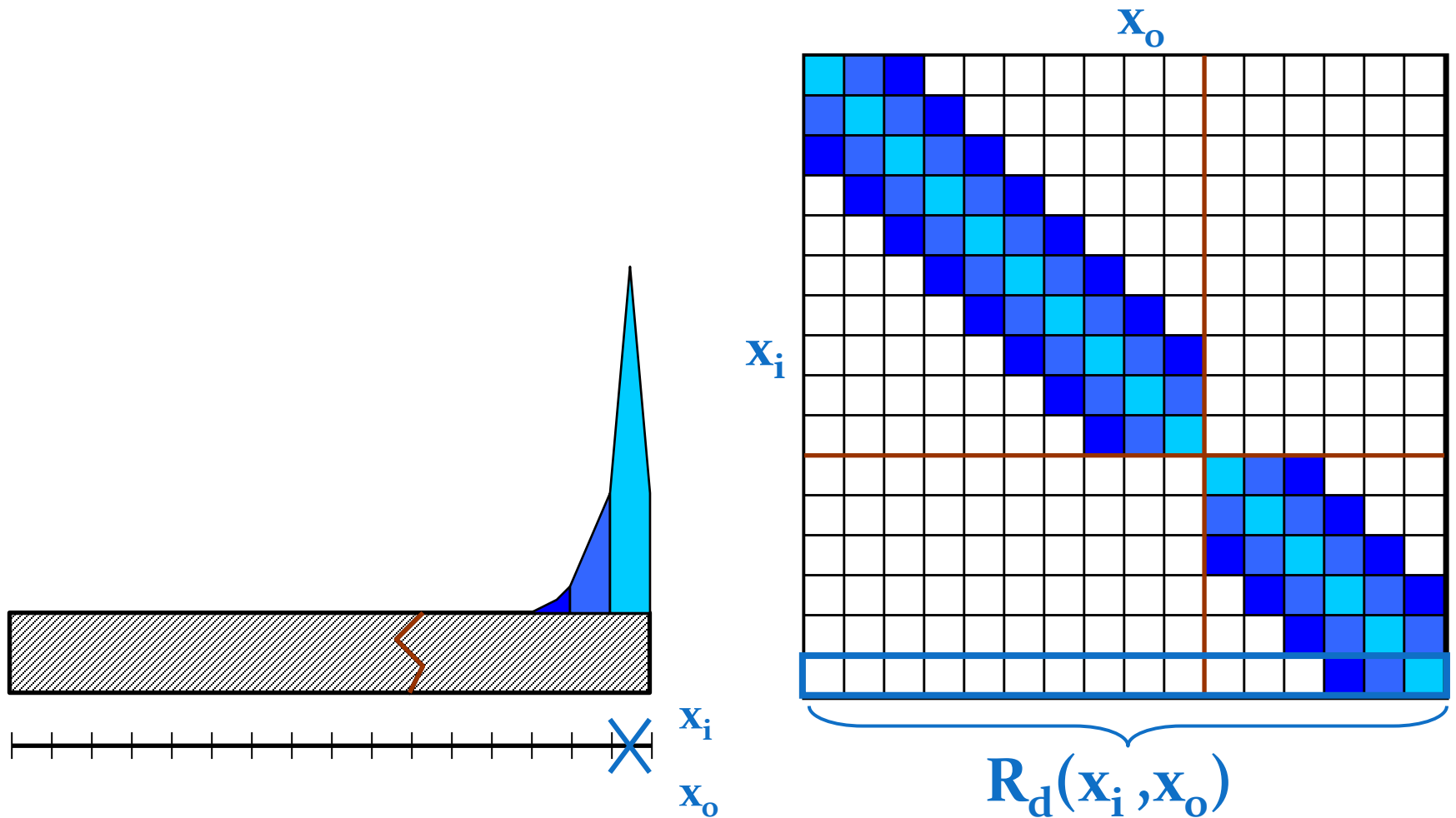
# Subsurface Scattering Matrix



# Subsurface Scattering Matrix



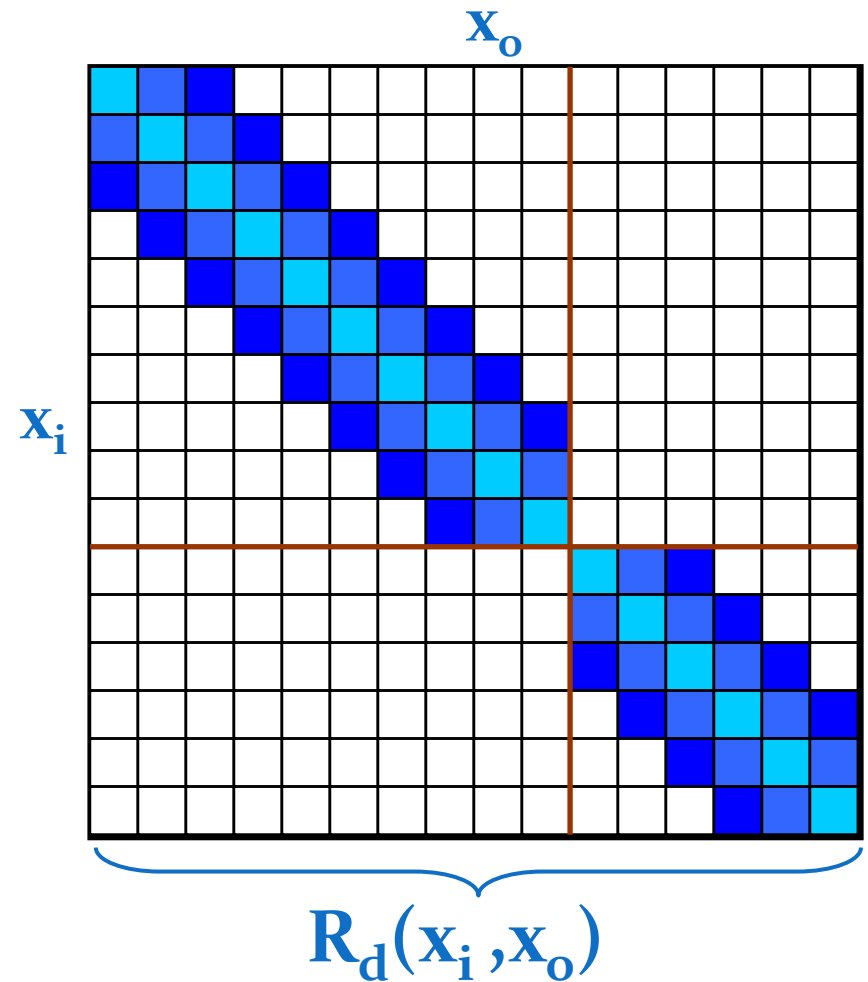
# Subsurface Scattering Matrix



Scanning all points, yields a matrix that looks something like this.



# Subsurface Scattering Matrix

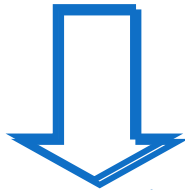


A few important observations can be made regarding the structure of this matrix.

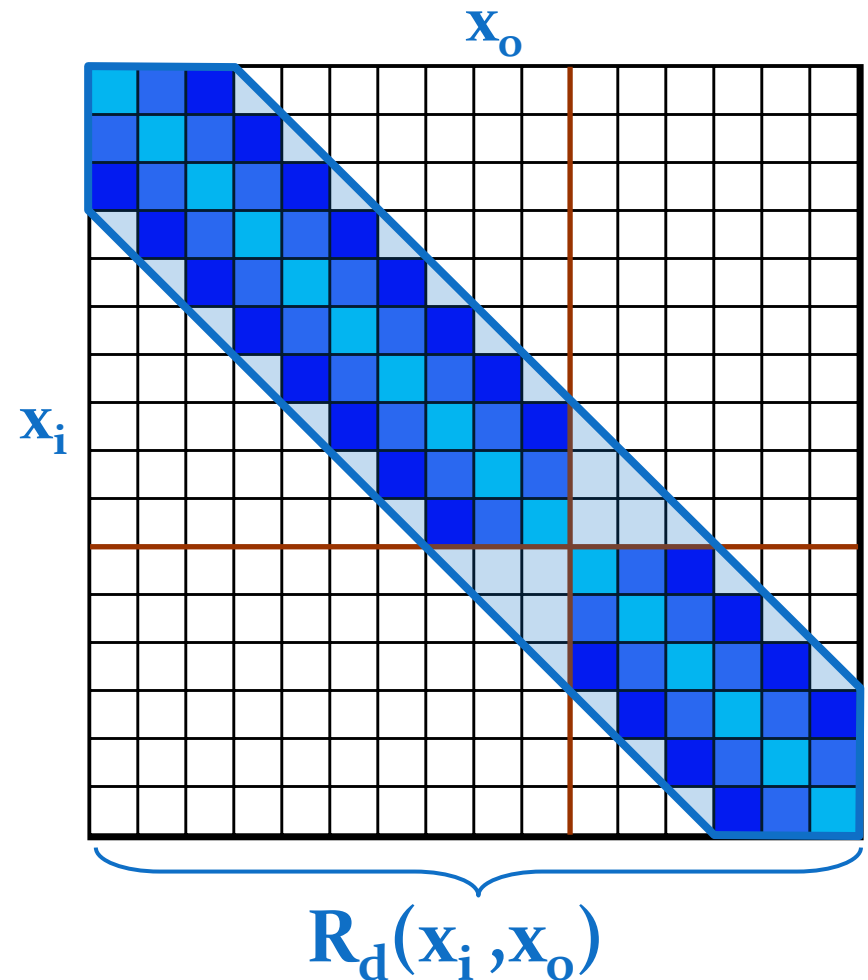


# Structure of Homogeneity

Homogeneous  
Scattering



Diagonal Band



First of all, it is a band diagonal matrix. Diagonal band is the result of the homogeneous subsurface scattering. This band is interrupted by...



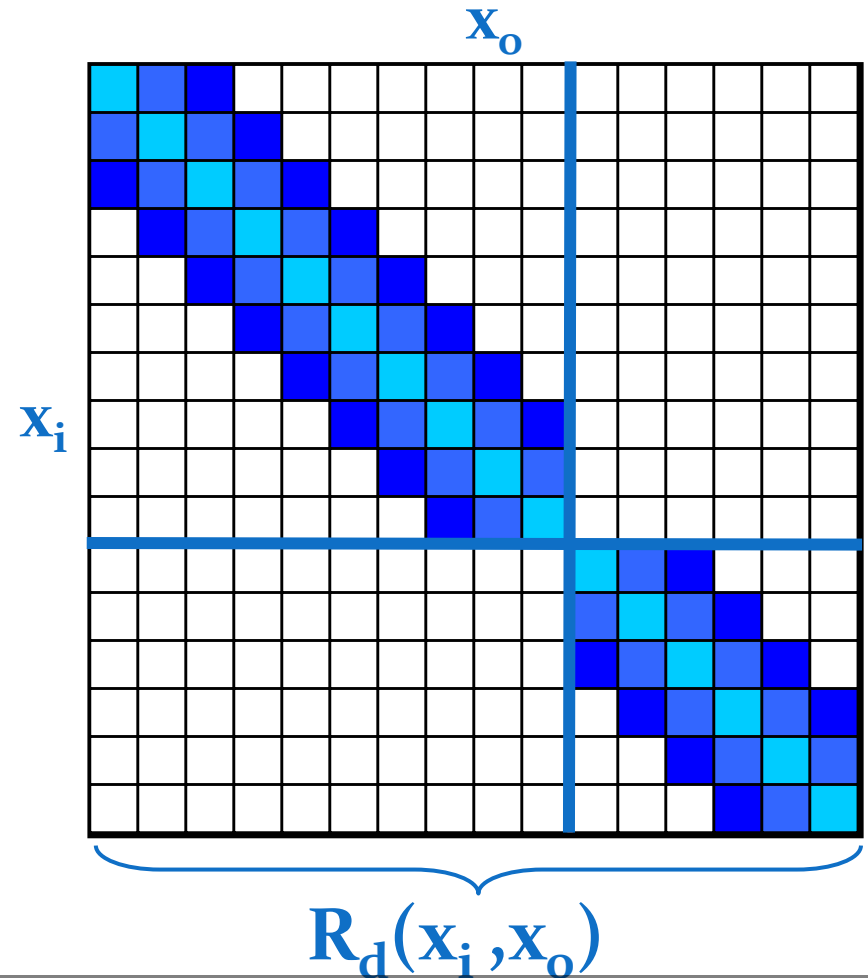


# Structure of Heterogeneities

Heterogeneities

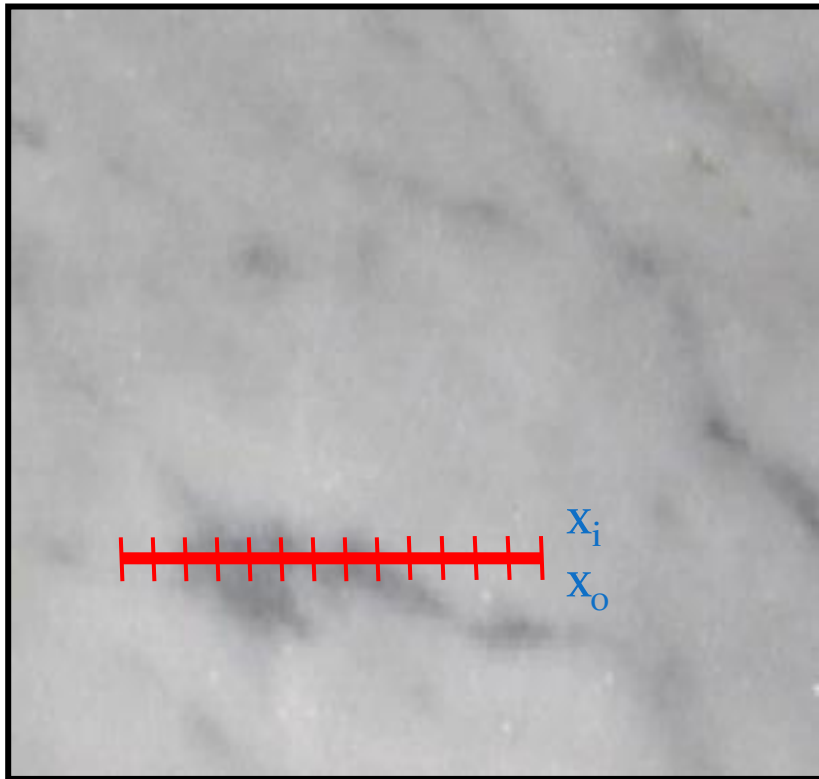


Horizontal &  
Vertical  
Discontinuities  
(in Matrix  $R_d$ )

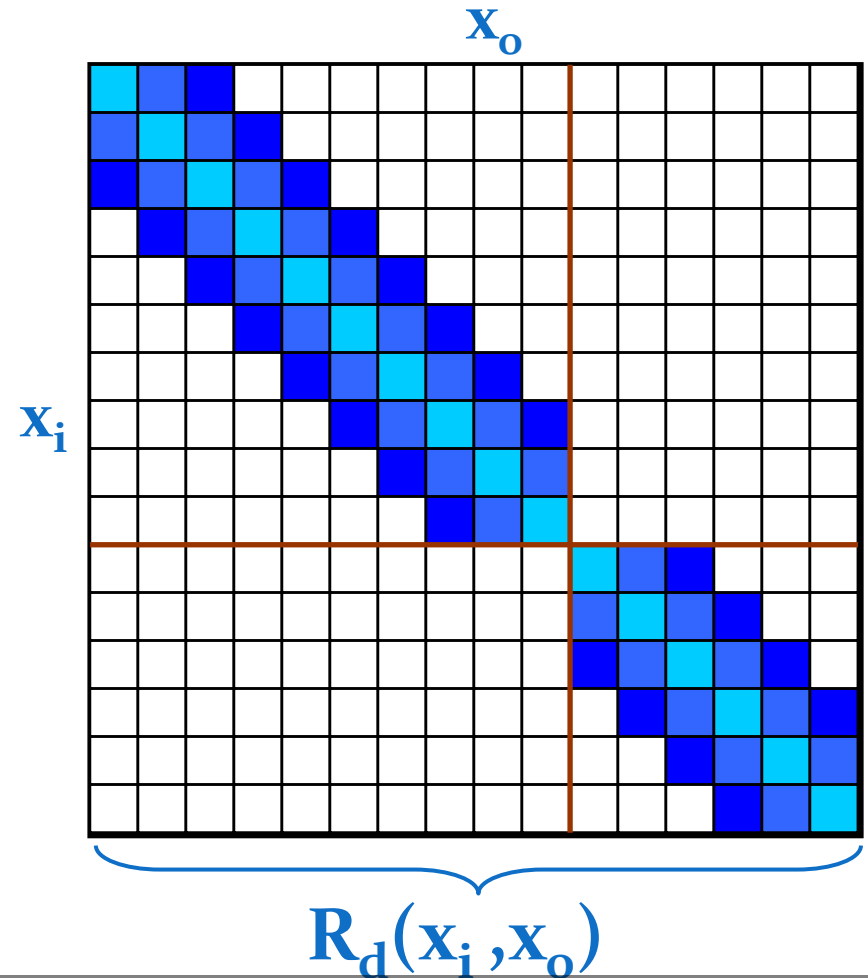


...horizontal and vertical discontinuities. These discontinuities in the matrix  $R_d$  are caused by the heterogeneity. Now, this all well for this synthetic example, but how well does this observed structure corresponds to a real subsurface scattering material.

# Validation of Structure

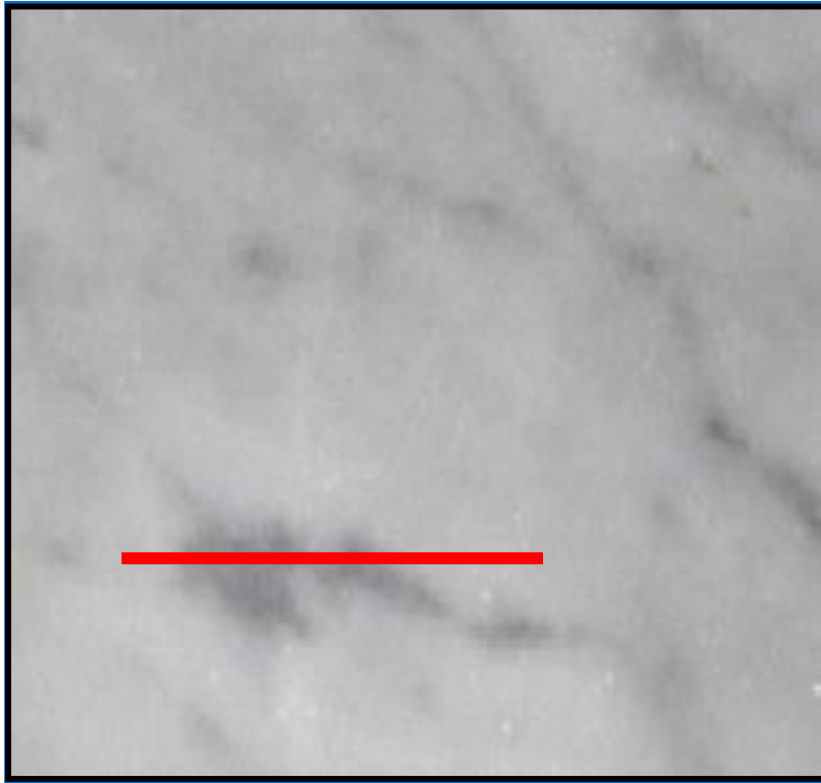


**Marble Sample**

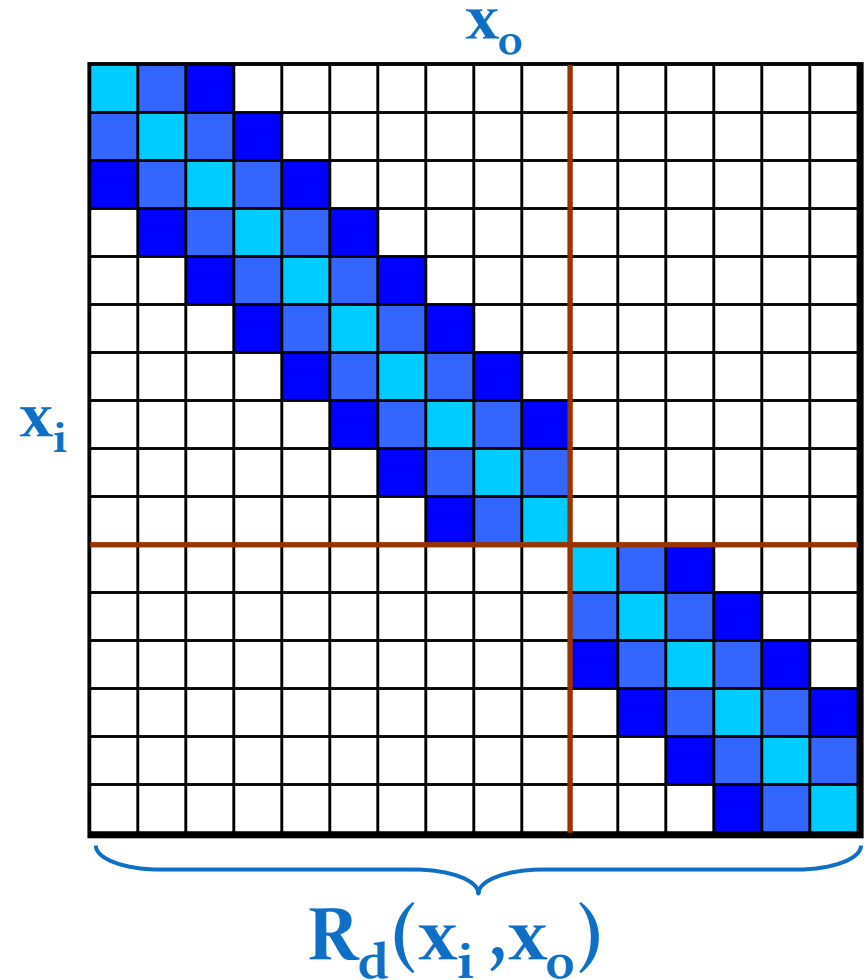


For this purpose we consider this slab of Marble. On this slab of marble, we are going to illuminate each point on this line <animate>, and are going to observe the response, also on this line.

# Validation of Structure



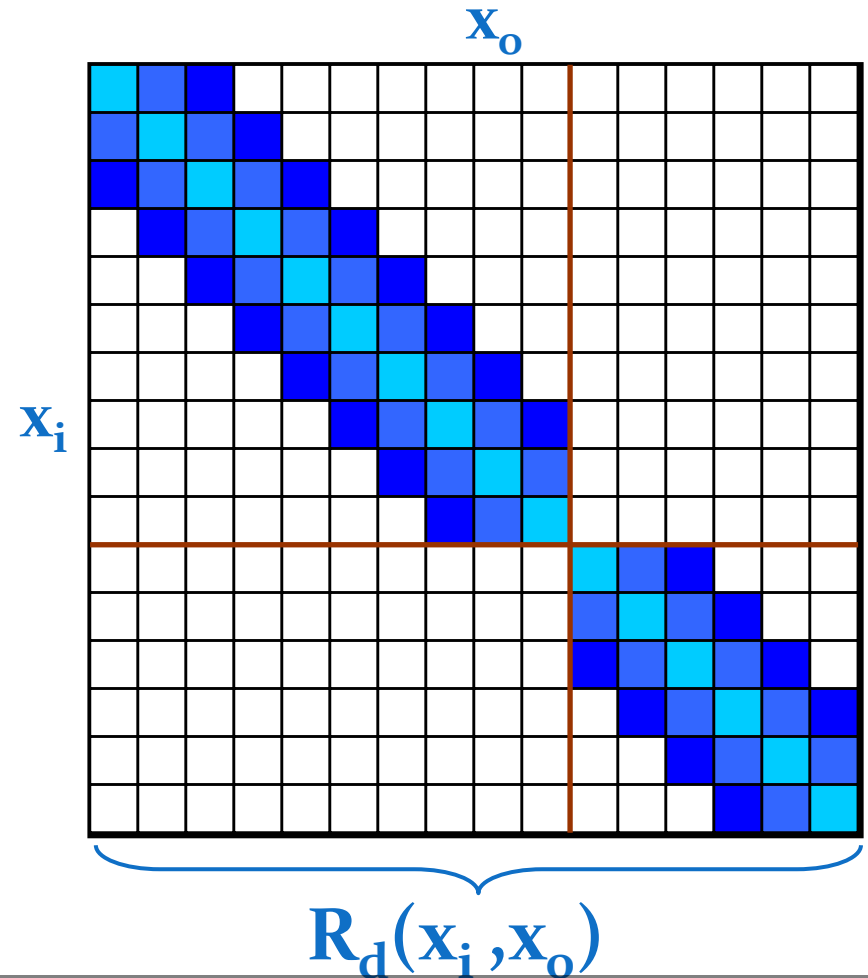
Marble:  $R_d(x_i, x_o)$



# Validation of Structure

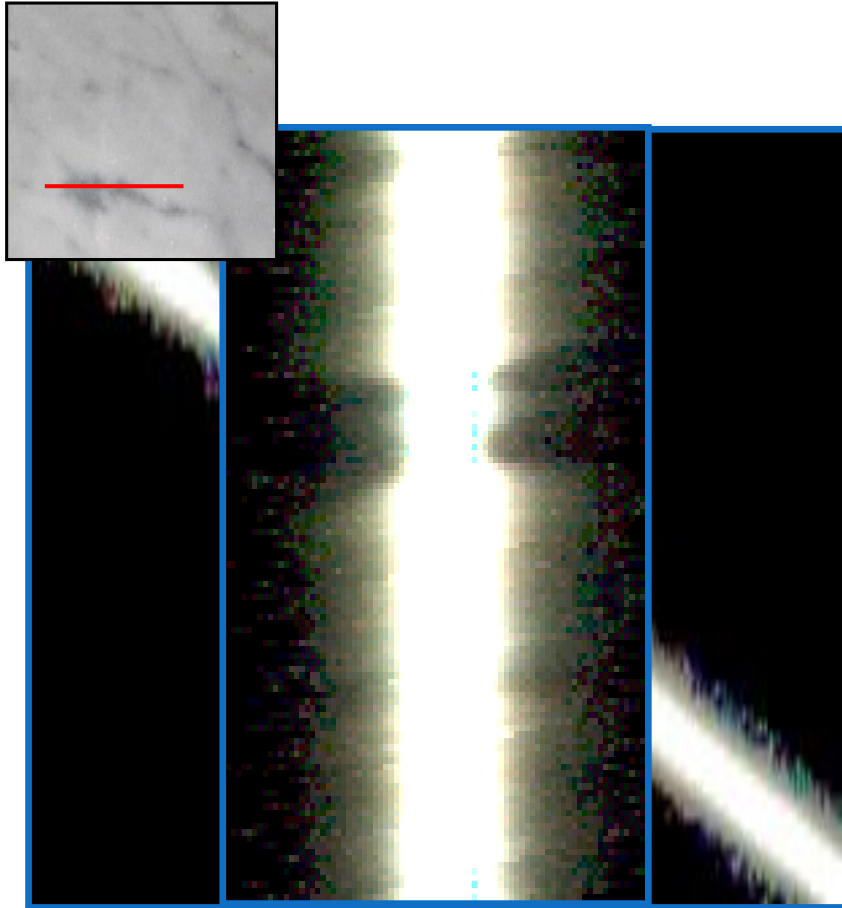


**Marble:**  $R_d(x_i, x_o)$

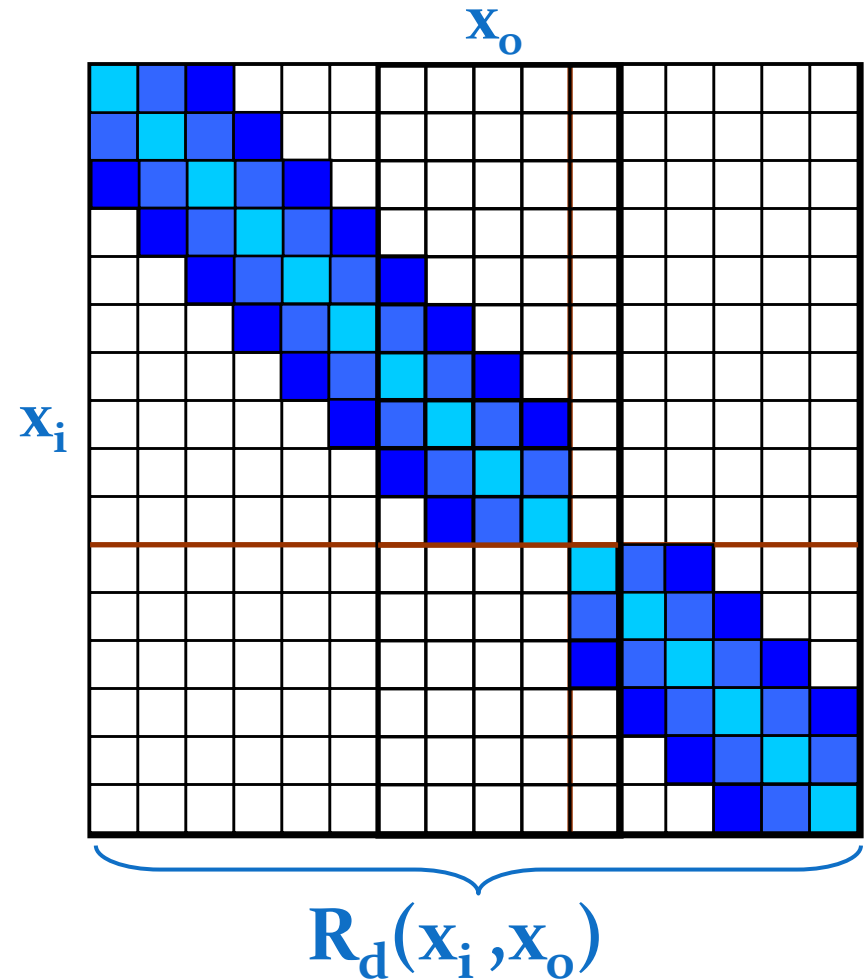


I don't know if you can see it, but the discontinuities are in the left top. The reason why these discontinuities are not as underlined as in the synthetic case, is because the veins in the marble do not block the light transport completely.

# Validation of Structure

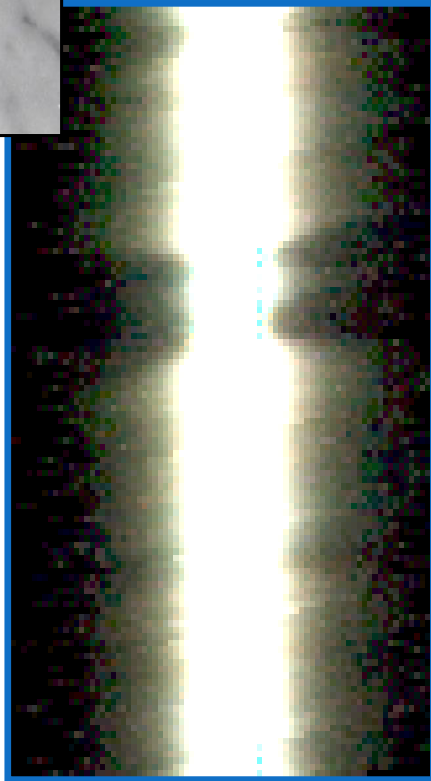


Marble:  $R_d(x_i, x_o)$

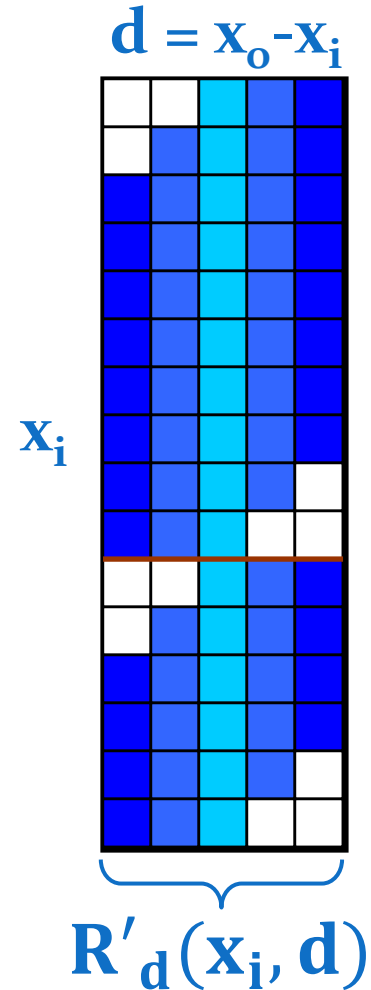


In order to better bring out these discontinuities, we are going to reparameterize the matrix  $R_d$  by shearing it, resulting in a matrix  $R_d$ -prime.

# Validation of Structure



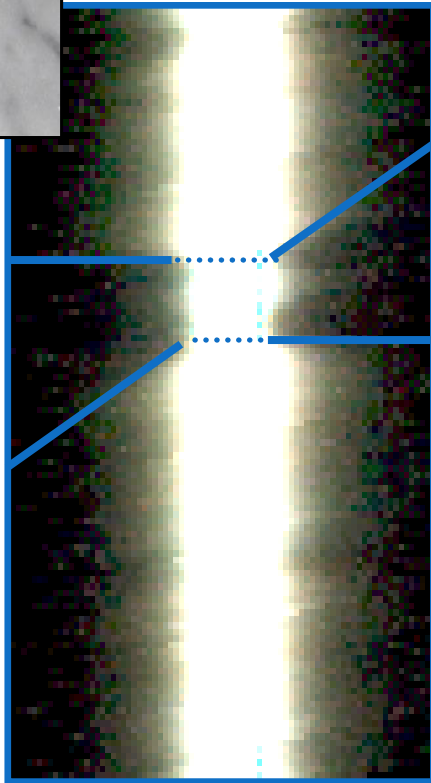
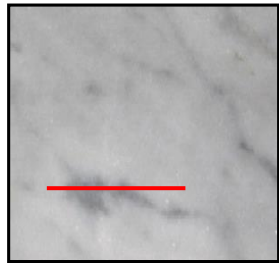
Marble:  $R'_d(\mathbf{x}_i, \mathbf{d})$



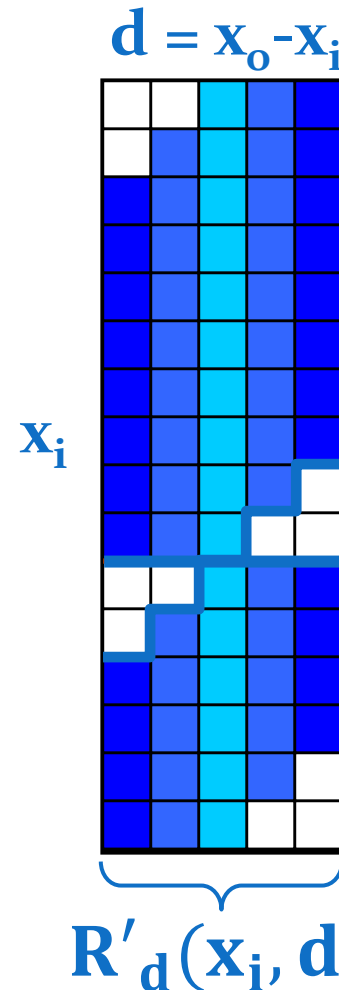
As you can see now, the discontinuities are much better visible.



# Validation of Structure

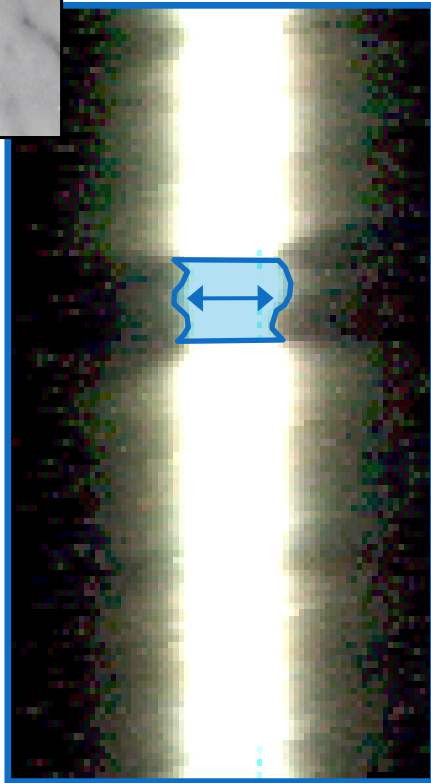
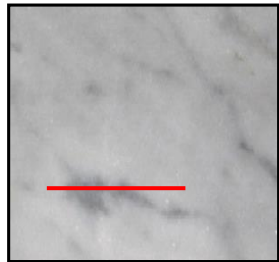


**Marble:**  $R'_d(\mathbf{x}_i, \mathbf{d})$

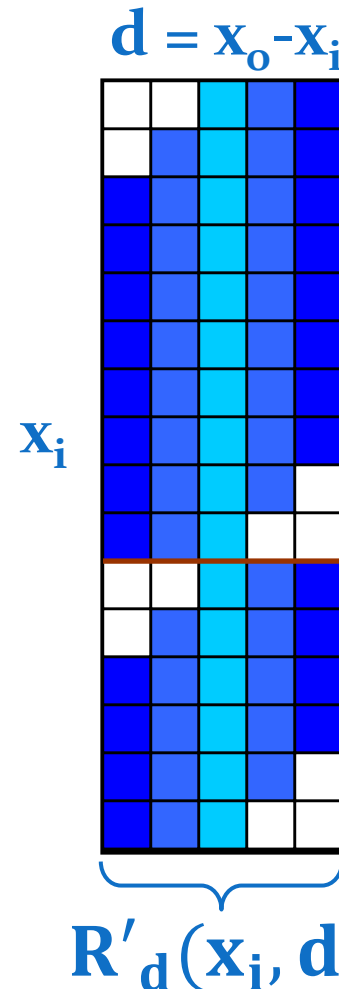


Another thing to notice is that the horizontal and vertical discontinuities have now been removed. This example, empirically validates our conclusions regarding the structure in the reparameterized subsurface scattering matrix of the marble sample.

# Validation of Structure



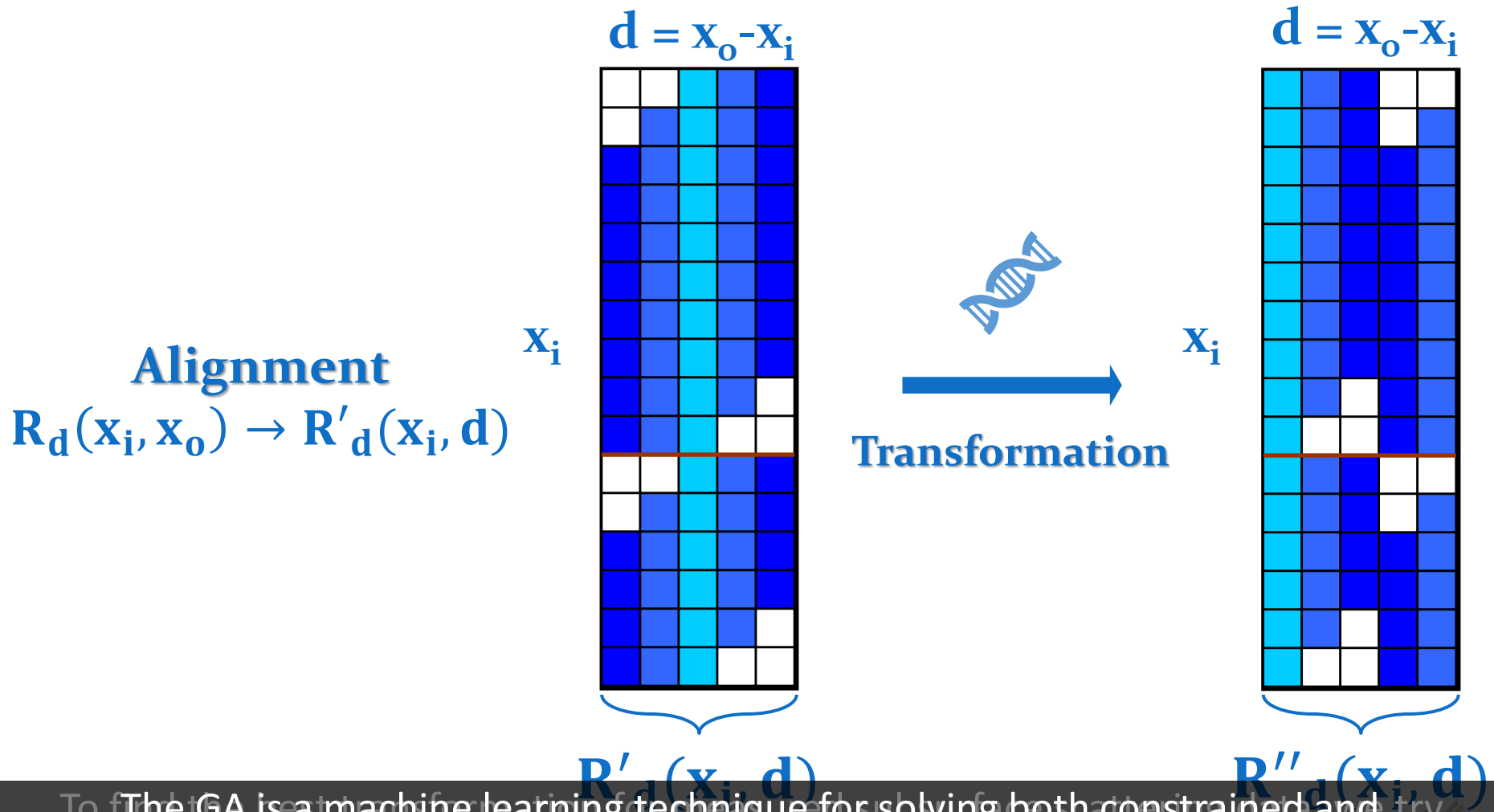
**Marble:  $R'_d(x_i, d)$**



Note that the width of this part in the matrix, is dependent on the amount of light blocked by the heterogeneity. Thus in case the veins would block all light, then we get a similar situation as in the synthetic example, and this region would be very thin.



# Key Idea for Subsurface Scattering Modeling



The GA is a machine learning technique for solving both constrained and unconstrained optimization problems by imitating biological evolution processes. We apply GA to find new transformations for real-world subsurface scattering.

# Our Genetic Algorithm (1)



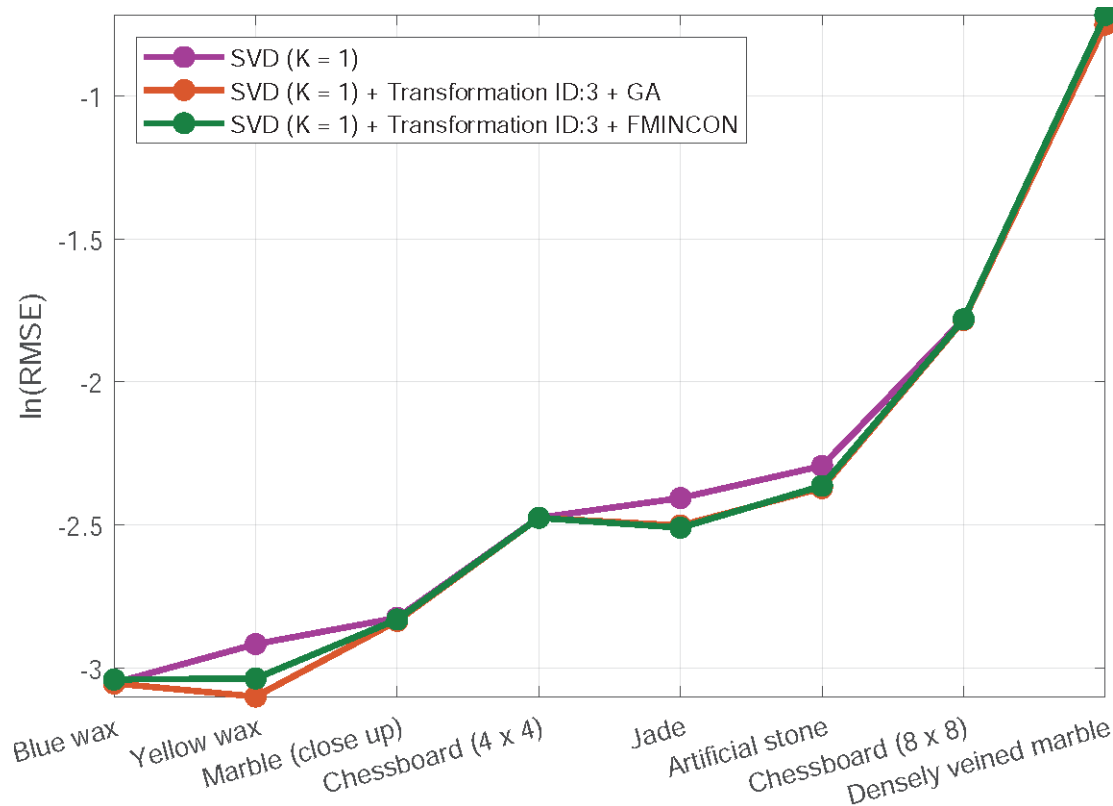
*Artificial stone material*

Transformation ID	Transformation expression	Chromosome	Population size	Fittest value (RMSE)
1	$R_d''(x_i, d) = \ln \left( 1 + \frac{R_d'(x_i, d)}{\alpha_s} \right)$	$\alpha_s(R, G, B)$	30	0.09694
2	$R_d''(x_i, d) = \ln \left( \alpha_d + \frac{R_d'(x_i, d)}{\alpha_s} \right)$	$\alpha_d(intensity), \alpha_s(intensity)$	20	0.09699
3	$R_d''(x_i, d) = \begin{cases} R_d'(x_i, d) / \max(R_d'(x_i, d)) & if range = 0 \\ \ln \left( 1 + \frac{R_d'(x_i, d)}{\alpha_s \max(R_d'(x_i, d))} \right) & otherwise \end{cases}$	$\alpha_s(R, G, B), d \pm range(R, G, B)$	60	0.09340

**Table 1:** Properties of the genetic optimization for heterogeneous artificial stone. The table also summarizes some statistics of transformations applied by our genetic algorithm with  $K = 1$ . When we don't apply any transformations, the rank-1 approximation of artificial stone gives RMSE of 0.1009.

Over successive generations, the population evolves toward an optimal solution based on a fitness function. At each step the algorithm selects the fittest individuals from the current population.

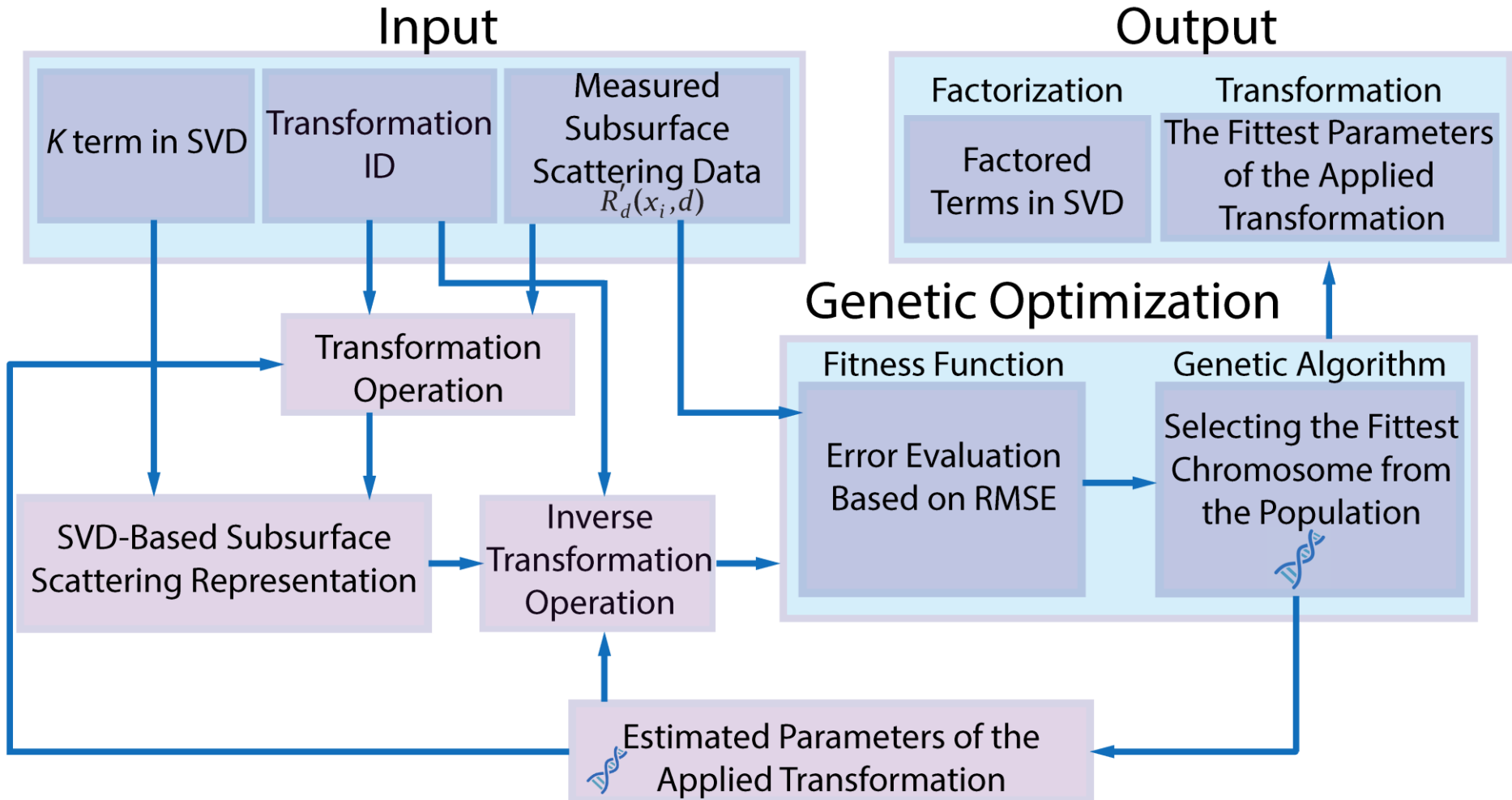
# Our Genetic Algorithm (2)



A comparison of the SVD based subsurface scattering model with and without applying various optimization techniques (see Table 1). The model parameter  $K$  was selected as 1. The error values were sorted in the logarithmic RMSEs of the SVD technique (purple) for visualization purposes.

On average, transformation ID:3 with GA decreases the RMSE much more than transformation ID:3 with FMINCON. Therefore, we select and use transformation ID:3 with our GA framework. These optimization techniques are readily available in MATLAB library.

# Our Genetic Optimization Framework



In this figure, you can see our genetic optimization framework. We investigate various transformation and chromosome combinations (see Table 1) thorough our genetic optimization framework

# Our Subsurface Scattering Model

$$\begin{array}{c}
 \mathbf{d} = \mathbf{x}_o - \mathbf{x}_i \\
 \mathbf{x}_i \\
 \mathbf{R}''_{\mathbf{d}}(\mathbf{x}_i, \mathbf{d}) \approx \underbrace{\left[ \mathbf{f}_1(\mathbf{x}_i) \underbrace{\left[ \underbrace{s_1}_{\text{scalar}} \underbrace{\mathbf{v}_1(\mathbf{d})}_{\text{vector}} \right]}_{\mathbf{h}_1(\mathbf{d}) = s_1 \mathbf{v}_1(\mathbf{d})} \right]}_{\text{K times}} + \dots + \underbrace{\left[ \mathbf{f}_K(\mathbf{x}_i) \underbrace{\left[ \underbrace{s_K}_{\text{scalar}} \underbrace{\mathbf{v}_K(\mathbf{d})}_{\text{vector}} \right]}_{\mathbf{h}_K(\mathbf{d}) = s_K \mathbf{v}_K(\mathbf{d})} \right]}
 \end{array}$$

$$\mathbf{R}''_{\mathbf{d}}(\mathbf{x}_i, \mathbf{d}) \approx \sum_{j=1}^K \mathbf{f}_j(\mathbf{x}_i) \mathbf{h}_j(\mathbf{d})$$

Our final subsurface scattering model will be the sum of the estimation of model errors and the first factorization of  $\mathbf{R}_{\mathbf{d}}$  prime prime.

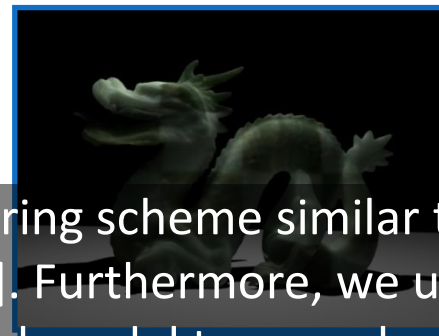
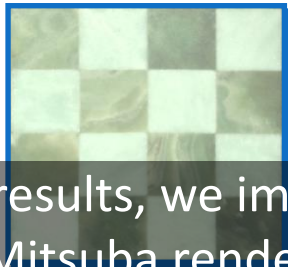


# Visualization

- *The full Monte Carlo path tracing implementation of MITSUBA [Jak13]:*
  - 1) *Blue noise sampling of translucent object.*
  - 2) *The weighting of all samples, within the range of the subsurface scattering response and adding together.*
  - 3) *The weighting function = the subsurface scattering representation.*
- *A standard texture mapping to apply a material model to a mesh.*

**Chessboard**

(4 × 4)



To visualize our results, we implemented a rendering scheme similar to Peers et al. [PvBM06] in the Mitsuba rendering system [Jak13]. Furthermore, we used a standard texture mapping to apply a material model to a mesh.

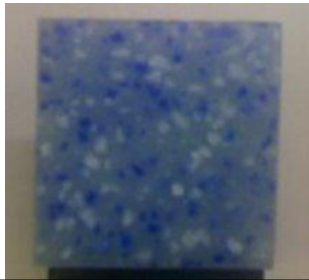
# Results – Validation

Sample material	Resolution (pixel)	Kernel size (pixel)	Original size	$K$	Factored size	CR	RMSE
Chessboard ( $4 \times 4$ )	$277 \times 277$	$39 \times 39$	2.61 GB	5	8.96 MB	1/298	0.0229
Chessboard ( $8 \times 8$ )	$222 \times 222$	$39 \times 39$	1.68 GB	5	5.82 MB	1/296	0.0421
Marble (close up)	$128 \times 128$	$39 \times 39$	570 MB	5	2.05 MB	1/278	0.0268
Densely veined marble	$213 \times 211$	$39 \times 39$	1.53 GB	5	5.32 MB	1/295	0.0568
Artificial stone	$108 \times 108$	$35 \times 35$	327 MB	5	1.48 MB	1/222	0.0340
Blue wax	$88 \times 232$	$35 \times 35$	572 MB	5	2.48 MB	1/231	0.0192
Jade	$260 \times 260$	$35 \times 35$	1.85 GB	5	7.88 MB	1/240	0.0398
Yellow wax	$110 \times 110$	$39 \times 39$	421 MB	5	1.56 MB	1/270	0.0225

**Table 2:** Properties of the factored heterogeneous subsurface scattering materials. The table also summarizes some statistics of our GA based subsurface scattering model with typically selected values for  $K$ .



Chessboard



Artificial stone



Blue wax



Jade

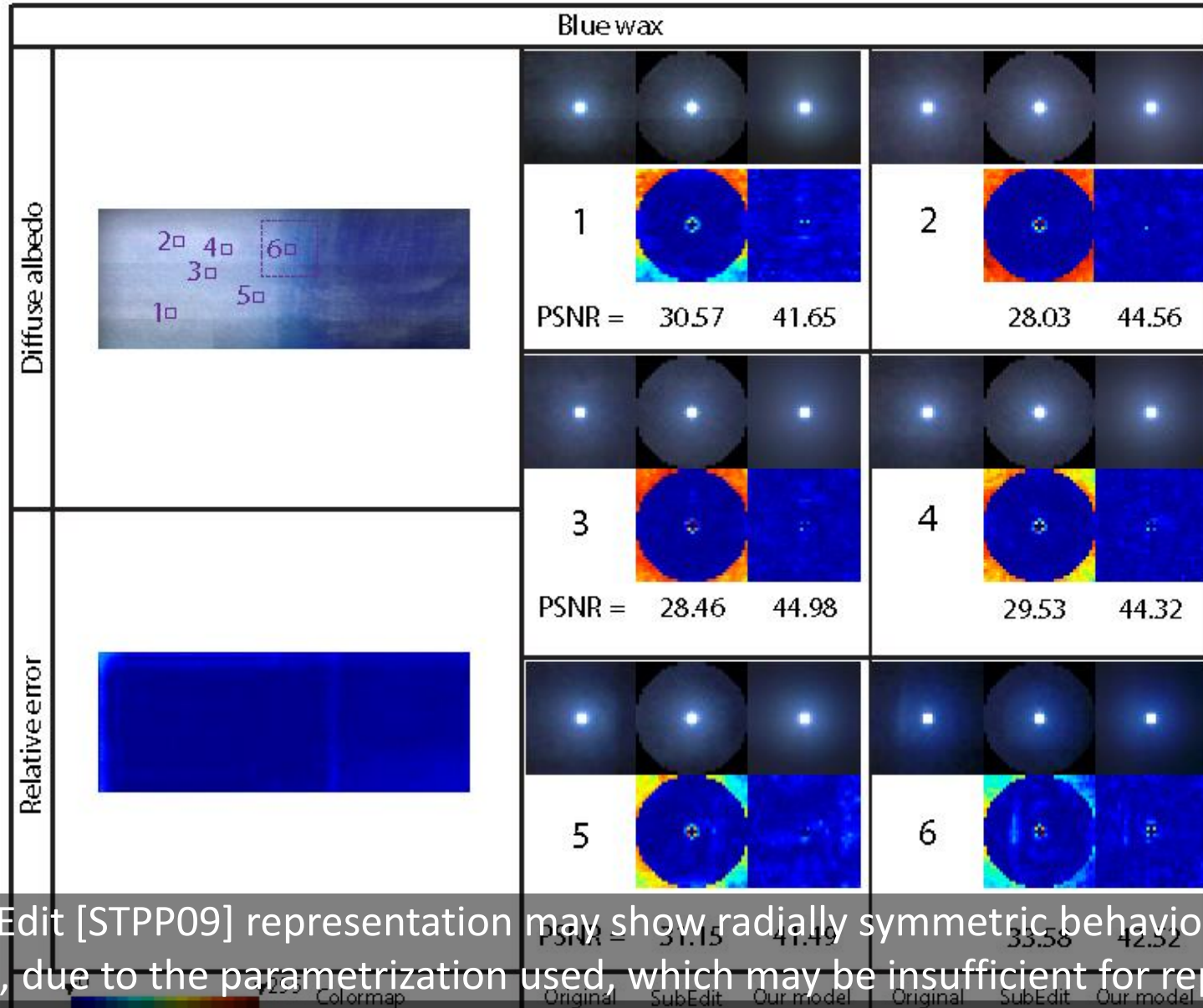


Yellow wax

We validate our model on 8 different real-world translucent materials. This table gives an overview of the modeled heterogeneous translucent materials and lists a number of statistics for our model, based on typical values for  $K$ .



# Results - Comparisons (1)



The SubEdit [STPP09] representation may show radially symmetric behavior at some materials, due to the parametrization used, which may be insufficient for representing heterogeneous materials accurately. We compare measured and modeled results for selected surface points.



# Results - Comparisons (2)



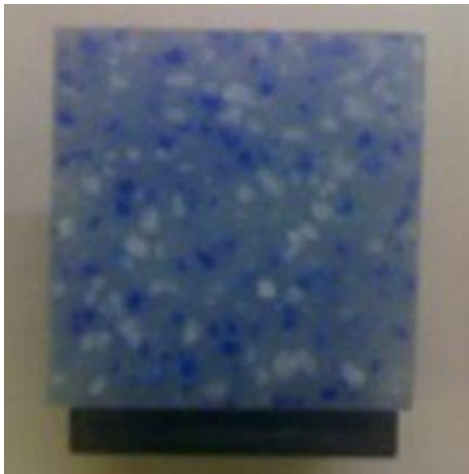
(Storage: 327 MB)



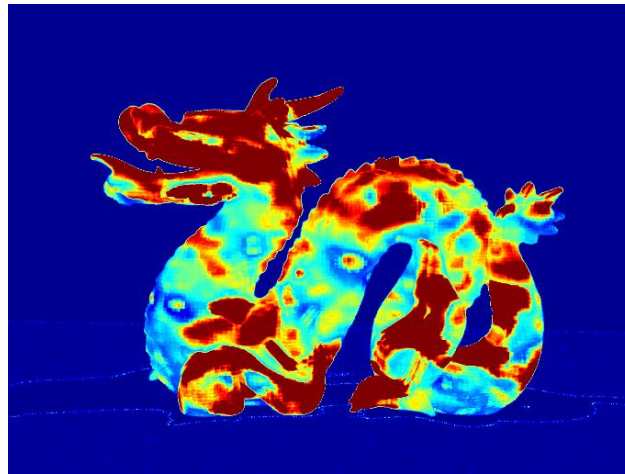
(Storage: 3.29 MB)



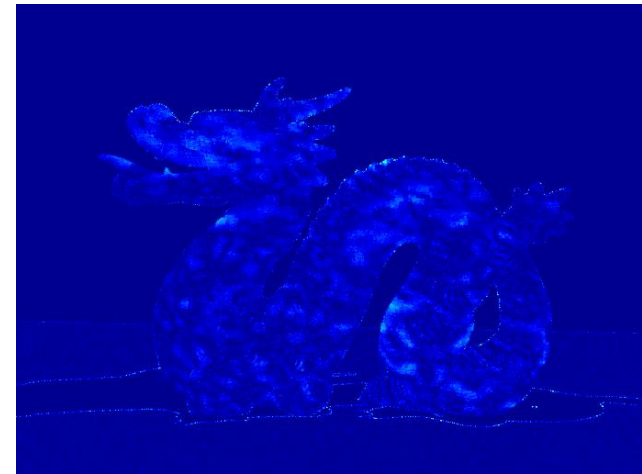
(Storage: 2.94 MB)



Reference image



SubEdit Model



Our Model (K=10)

The comparisons outlined show that our model represents heterogeneous translucent materials more accurately for comparable data storage requirements.

# Conclusions

- A GA based heterogeneous subsurface scattering model.
- *Apply to any geometry.*
- *Integrate into a standard physically-based rendering system easily.*
- We plan to investigate real-time rendering algorithms to implement our representation in screen-space.

In the future, we plan to investigate real-time rendering algorithms to implement our

In this work, we present a GA based heterogeneous subsurface scattering model.

# Thank You...

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I hope you enjoyed this presentation, and I thank you for your attention.